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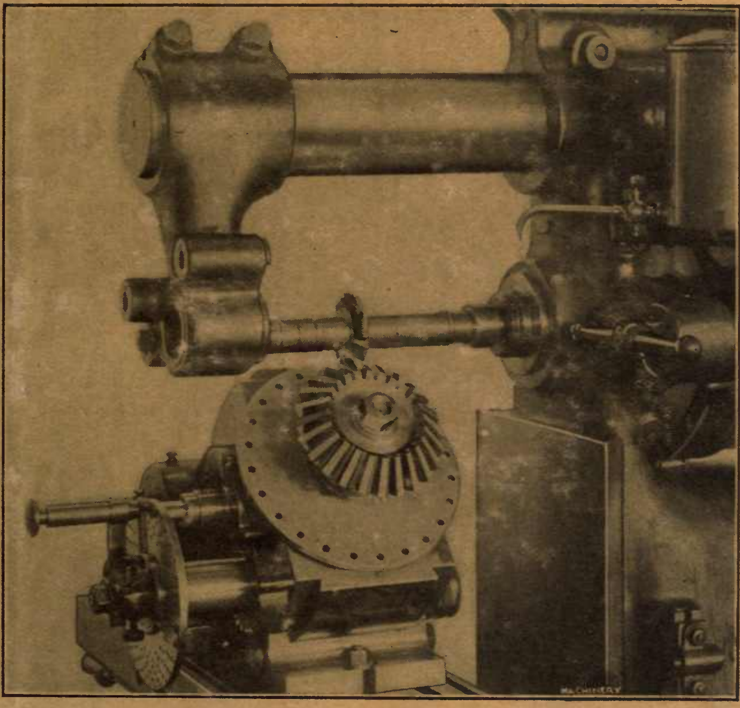
BEVEL GEARING

CALCULATION—DESIGN—CUTTING THE TEETH

BY RALPH E. FLANDERS

FOURTH REVISED EDITION



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NUMBER 37

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CHAPTER I

BEVEL GEAR RULES AND FORMULAS

Bevel gearing, as every mechanic knows, is the form of gearing used for transmitting motion between shafts whose center lines intersect. The teeth of bevel gears are constructed on imaginary pitch cones in the same way that the teeth of spur gears are constructed on imaginary pitch cylinders. In Fig. 1 is shown a drawing of a pair of bevel gears of which the gear has twice as many teeth as the pinion. The latter thus revolves twice for every revolution of the gear. In Fig. 2 is shown (diagrammatically) a pair of conical pitch surfaces driving each other by frictional contact. The shafts are set at the same center angle with each other, as in Fig. 1, and the base diameter of the gear cone is twice that of the pinion cone, so that the latter will revolve twice to each revolution of the former. This being the case, the cones shown in Fig. 2 are the pitch cones of the gears shown in

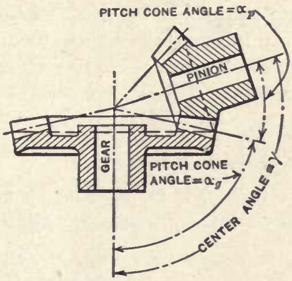


Fig. 1. Bevel Gear and Pinion

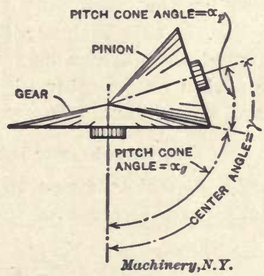


Fig. 2. Pitch Cones of Gears Shown in Fig. 1

Fig. 1. We may therefore define the term "pitch cone" as follows: The pitch cones of a pair of bevel gears are those cones which, when mounted on the shafts in place of the bevel gears, will drive each other by frictional contact in the same velocity ratio as given by the bevel gears themselves.

The pitch cones are defined by their pitch cone angles, as shown in Fig. 2. The sum of the two pitch cone angles equals the center angle, the latter being the angle made by the shafts with each other, measured on the side on which the contact between the cones takes place. The center angle and the pitch cone angles of the gear and the pinion are indicated in Fig. 1.

Different Kinds of Bevel Gears

In Fig. 3 is shown a pair of bevel gears in which the center angle (γ) equals 90 degrees, or in other words, the figure shows a case of right angle bevel gearing. To the special case shown in Fig. 4 in which the number of teeth in the two gears is the same, the term miter gearing is applied; here the pitch cone angle of each gear will always equal 45 degrees.

When the pitch cone angle is less than 90 degrees we have acute angle bevel gearing, as shown in Fig. 5. When the center angle is greater than 90 degrees, we have obtuse angle bevel gearing, shown in Fig. 6 and also in Fig. 1. Obtuse angle bevel gearing is met with occasionally in the two special forms shown in Figs. 7 and 8. When the pitch cone angle α_g equals 90 degrees, the gear g is called a crown gear. In this case the pitch cone evidently becomes a pitch plane, or

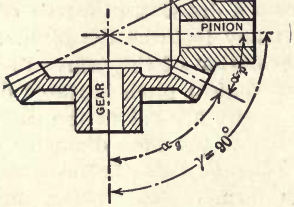


Fig. 3. Right Angle Bevel Gearing

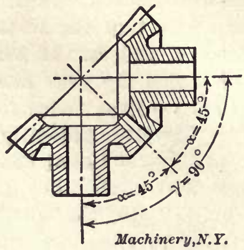


Fig. 4. Miter Gearing

disk. When the pitch cone angle of the gear is more than 90 degrees, as in Fig. 8, this member is called an internal bevel gear, and its pitch cone when drawn as for Fig. 2, would mesh with the pitch cone of the pinion on its internal conical surface. These two special forms of gears are of rare occurrence.

Bevel Gear Dimensions and Definitions*

In Fig. 9, which shows an axial section of a bevel gear, the pitch lines show the location of the periphery of the imaginary pitch cone.

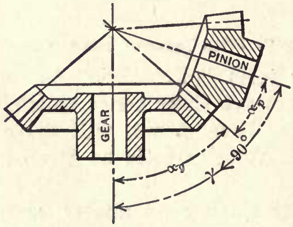


Fig. 5. Acute Angle Bevel Gearing

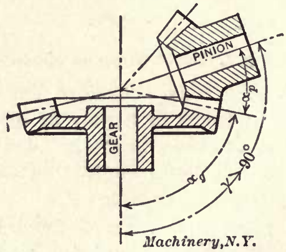


Fig. 6. Obtuse Angle Bevel Gearing

The pitch cone angle is the angle which the pitch line makes with the axis of the gear. The pitch diameter is measured across the gear drawing at the point where the pitch lines intersect the outer edge of the teeth. The teeth of bevel gears grow smaller as they approach the vertex *O* of the pitch cone, where they would disappear if the teeth were cut for the full length of the face. In speaking of the pitch of a bevel gear we always mean the pitch of the larger or outer ends of the teeth. Diametral and circular pitch have the same meaning as in the case of spur gears, the diametral pitch being the num-

*MACHINERY, February, 1910.

ber of teeth per inch of the pitch diameter, while the circular pitch is the distance from the center of one tooth to the center of the next, measured along the pitch diameter at the back faces of the teeth. The addendum is the height of the tooth above the pitch line at the large end. The dedendum (the depth of the tooth space below the pitch line) and the whole depth of the tooth are also measured at the large end.

The pitch cone radius is the distance measured on the pitch line from the vertex of the pitch cone to the outer edge of the teeth. The width of the face of the teeth, as shown in Fig. 9, is measured on a line parallel to the pitch line. The addendum, whole depth and thickness of the teeth at the small or inner end may be derived from the corresponding dimensions at the outer end, by calculations depending on the ratio of the width of face to the pitch cone radius. (See *s*, *w* and *t* in Fig. 12.)

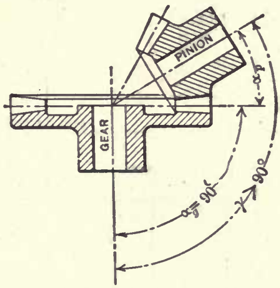


Fig. 7. Crown Gear and Pinion

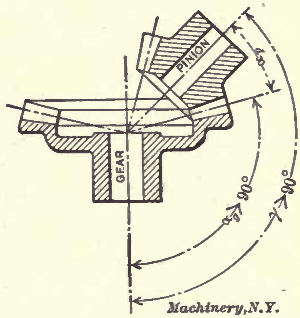


Fig. 8. Internal Bevel Gear and Pinion

The addendum angle is the angle between the top of the tooth and the pitch line. The dedendum angle is the angle between the bottom of the tooth space and the pitch line. The face angle is the angle between the top of the tooth and a perpendicular to the axis of the gear. The edge angle (which equals the pitch cone angle) is the angle between the outer edge and the perpendicular to the axis of the gear. The latter two angles are measured from the perpendicular instead of from the axis, for the convenience of the workman in making measurements with the protractor when turning the blanks. The cutting angle is the angle between the bottom of the tooth space and the axis of the gear.

The angular addendum is the height of tooth at the large end above the pitch diameter, measured in a direction perpendicular to the axis of the gear. The outside diameter is measured over the corners of the teeth at the large end. The vertex distance is the distance measured in the direction of the axis of the gear from the corner of the teeth at the large end to the vertex of the pitch cone. The vertex distance at the small end of the tooth is similarly measured.

The shape of the teeth of a bevel gear may be considered as being the same as for teeth in a spur gear of the same pitch and style of tooth, having a radius equal to the distance from the pitch line at the back edge of the tooth to the axis of the gear, measured in a direction perpendicular to the pitch line. This distance is dimensioned $\frac{D'}{2}$ in Fig. 12. The number of teeth which such a spur gear would have, as determined by diameter D' thus obtained, may be called the "number of teeth in equivalent spur gear," and is used in selecting the cutter for forming the teeth of bevel gears by the formed cutter process.

In two special forms of gears, the crown gear, Fig. 10, and the internal bevel gear, Fig. 11, the same dimensions and definitions apply as in regular bevel gears, though in a modified form in some cases. In the crown gear, for instance, the pitch diameter and the outside diameter are the same, and the pitch cone radius is equal to ½ the pitch diameter. The addendum angle and the face angle are also the same. The angular addendum becomes zero, and the vertex distance is equal to the addendum. The number of teeth in the equivalent spur gear becomes infinite, or in other words, the teeth are shaped like those of a rack.

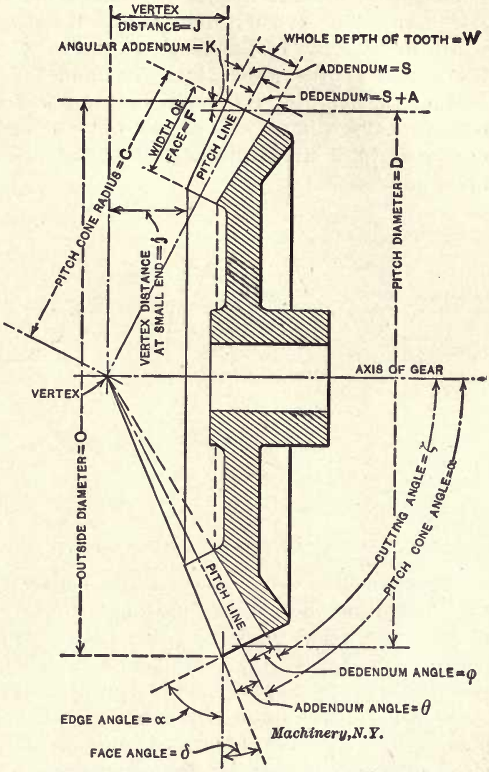


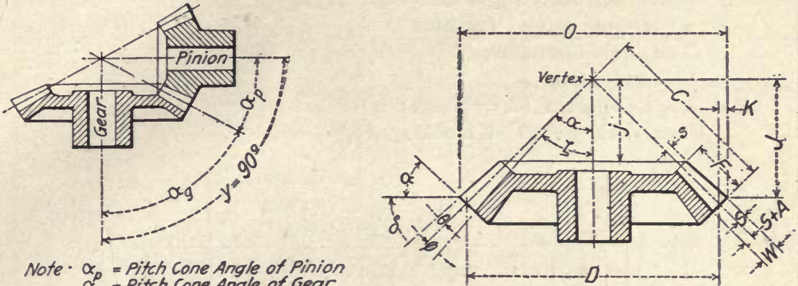
Fig. 9. Dimensions, Definitions and Reference Letters for Ordinary Bevel Gear

When the pitch cone angle is greater than 90 degrees, so that the gear becomes an internal bevel gear, as in Fig. 11, the outside diameter (or edge diameter as it is better called in the case of internal gears) becomes less than the pitch diameter. Otherwise the conditions are the same although many of the dimensions are reversed in direction.

Rules and formulas for calculating the dimensions of bevel gears are given on pages 7, 9, 11, and 13. The following reference letters are used:

CHART FOR SOLUTION OF BEVEL GEAR PROBLEMS.—I

Bevel Gears with Shafts at Right Angles.



Note · α_p = Pitch Cone Angle of Pinion
α_g = Pitch Cone Angle of Gear
N_p = Number of Teeth in Pinion, etc.

Use Rules and Formulas 1-21 in the order given.

No.	To Find		Rule	Formula
1	Pitch Cone Angle (or Edge Angle) of Pinion		Divide the number of teeth in the pinion by the number of teeth in the gear to get the tangent	$\text{Tan}\ \alpha_p = \frac{N_p}{N_g}$
2	Pitch Cone Angle (or Edge Angle) of Gear		Divide the number of teeth in the gear by the number of teeth in the pinion to get the tangent	$\text{Tan}\ \alpha_g = \frac{N_g}{N_p}$
3	Proof of Calculations for Pitch Cone Angles		The sum of the pitch cone angles of the pinion and gear equals 90 degrees	$\alpha_p + \alpha_g = 90°$
4	Pitch Diameter		Divide the number of teeth by the diametral pitch; or multiply the number of teeth by the circular pitch and divide by 3.1416	$D = \frac{N}{P} = \frac{NP'}{\pi}$
5	These dimensions are the same for both gear and pinion	Addendum	Divide 1.0 by the diametral pitch; or multiply the circular pitch by 0.318	$S = \frac{1.0}{P} = 0.318P'$
6		Dedendum	Divide 1.157 by the diametral pitch; or multiply the circular pitch by 0.368	$S+A = \frac{1.157}{P} = 0.368P'$
7		Whole Depth of Tooth Space	Divide 2.157 by the diametral pitch; or multiply the circular pitch by 0.687	$W = \frac{2.157}{P} = 0.687P'$
8		Thickness of Tooth at Pitch Line	Divide 1.571 by the diametral pitch; or divide the circular pitch by 2	$T = \frac{1.571}{P} = \frac{P'}{2}$
9		Pitch Cone Radius	Divide the pitch diameter by twice the sine of the pitch cone angle	$C = \frac{D}{2 \times \text{Sin}\ \alpha}$
10		Addendum at Small End of Tooth	Subtract the width of face from the pitch cone radius, divide the remainder by the pitch cone radius and multiply by the addendum	$s = S \times \frac{C-F}{C}$
11		Thickness of Tooth at Pitch Line at Small End	Subtract the width of face from the pitch cone radius, divide the remainder by the pitch cone radius and multiply by the thickness of the tooth at the pitch line	$t = T \times \frac{C-F}{C}$
12		Addendum Angle	Divide the addendum by the pitch cone radius to get the tangent	$\text{Tan}\ \theta = \frac{S}{C}$
13		Dedendum Angle	Divide the dedendum by the pitch cone radius to get the tangent	$\text{Tan}\ \varphi = \frac{S+A}{C}$

N = number of teeth,
P = diametral pitch,
P' = circular pitch,
π = 3.1416, (pi),
α = pitch cone angle and edge angle, (alpha),
γ = center angle, (gamma),
D = pitch diameter,
S = addendum,
$S + A$ = dedendum (A = clearance),
W = whole depth of tooth space,

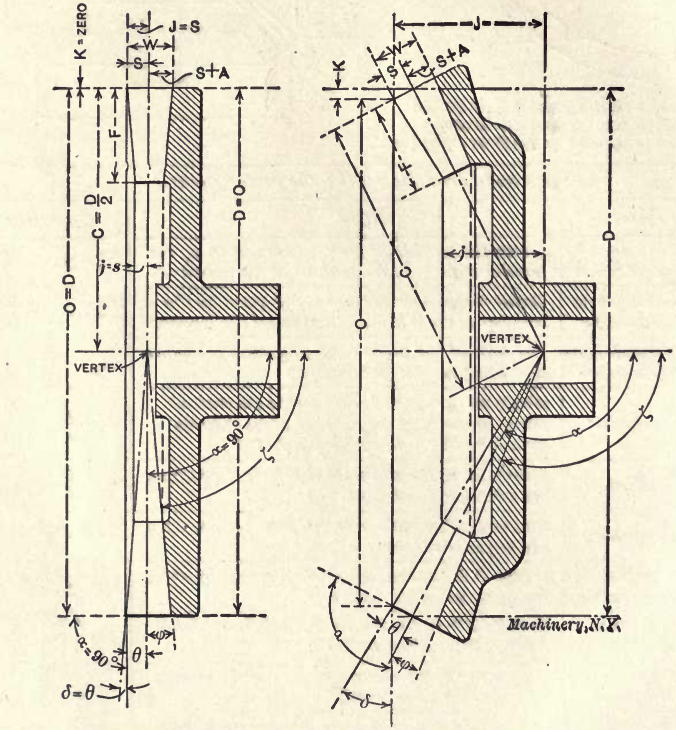


Fig. 10. Dimensions for Crown Gear Fig. 11. Dimensions for Internal Bevel Gear

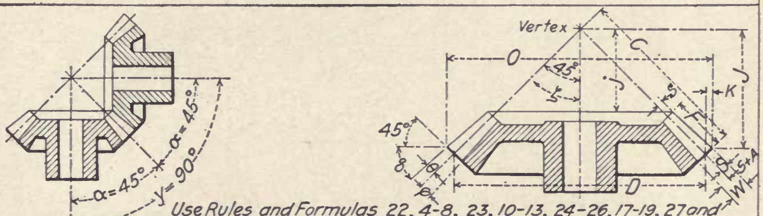
T = thickness of tooth at pitch line,
C = pitch cone radius,
F = width of face,
s = addendum at small end of tooth,
t = thickness of tooth at pitch line at small end,
θ = addendum angle, (theta),
ϕ = dedendum angle, (phi),
δ = face angle, (delta),
ζ = cutting angle, (zeta),
K = angular addendum,

CHART FOR SOLUTION OF BEVEL GEAR PROBLEMS.—II

Bevel Gears with Shafts at Right Angles (Continued).

No.	To Find	Rule	Formula
14	Face Angle	Subtract the sum of the pitch cone and addendum angles from 90 degrees	$\delta = 90° - (\alpha + \theta)$
15	Cutting Angle*	Subtract the dedendum angle from the pitch cone angle	$\zeta = \alpha - \varphi$
16	Angular Addendum	Multiply the addendum by the cosine of the pitch cone angle	$K = S \times \cos\alpha$
17	Outside Diameter	Add twice the angular addendum to the pitch diameter	$O = D + 2K$
18	Apex Distance	Multiply one-half the outside diameter by the tangent of the face angle	$J = \frac{O}{2} \times \tan\delta$
19	Apex Distance at Small End of Tooth	Subtract the width of face from the pitch cone radius; divide the remainder by the pitch cone radius and multiply by the apex distance	$j = J \times \frac{C-F}{C}$
20	Number of Teeth in Equivalent Spur Gear	Divide the number of teeth by the cosine of the pitch cone angle	$N' = \frac{N}{\cos\alpha}$
21	Proof of Calculations by Rules Nos. 9, 12, 14, 16 and 17	The outside diameter equals twice the pitch cone radius multiplied by the cosine of the face angle and divided by the cosine of the addendum angle	$O = \frac{2C \times \cos\delta}{\cos\theta}$

Mitre Bevel Gearing.



Use Rules and Formulas 22, 4-8, 23, 10-13, 24-26, 17-19, 27 and 21 in the order given. All dimensions thus obtained are the same for both gears of a pair

No.	To Find	Rule	Formula
22	Pitch Cone Angle	Pitch cone angle equals 45 degrees	$\alpha = 45°$
23	Pitch Cone Radius	Multiply the pitch diameter by 0.707	$C = 0.707D$
24	Face Angle	Subtract the addendum angle from 45°	$\delta = 45° - \theta$
25	Cutting Angle*	Subtract the dedendum angle from 45 degrees	$\zeta = 45° - \varphi$
26	Angular Addendum	Multiply the addendum by 0.707	$K = 0.707\,S$
27	Number of Teeth in Equivalent Spur Gear	Multiply the number of teeth by 1.41	$N' = 1.41N$

*For gears whose teeth are to be milled, see recommendation of the Brown & Sharpe Mfg. Co., Page 41.

$O =$ outside diameter (edge diameter for internal gears),
$J =$ vertex distance,
$j =$ vertex distance at small end,
$N' =$ number of teeth in equivalent spur gear.

Sub$_p$ refers to dimensions applying to pinion (a_p, N_p, etc.)
Sub$_g$ refers to dimensions applying to gear (a_g, N_g, etc.)

It will be noted that directions for the use of these rules are given for each of the six cases of right angle bevel gearing, miter bevel gearing, acute angle and obtuse angle bevel gearing, and crown and

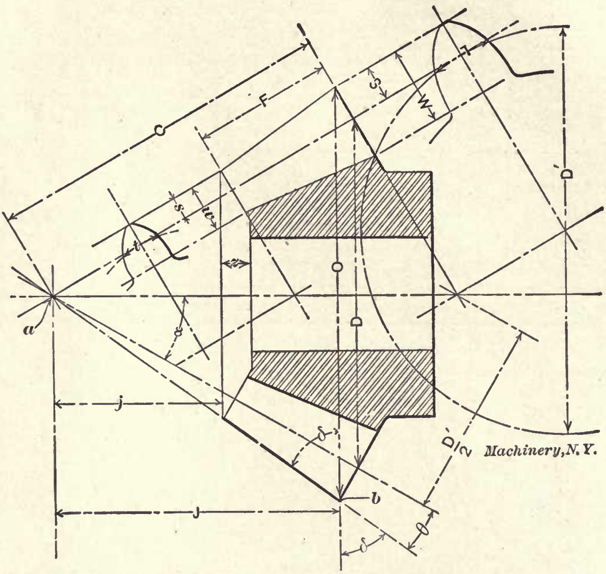


Fig. 12. Diagram Explaining Certain Calculations Relating to Bevel Gears

internal bevel gears. Further instruction as to their use can be obtained from the examples given in Chapter II.

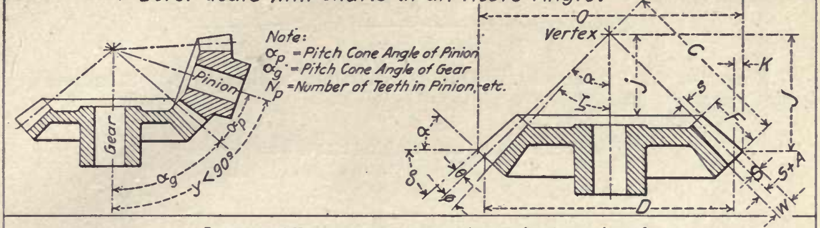
Rules and Formulas for Bevel Gear Calculations

The derivation of most of these formulas is evident on inspection of Figs. 1 to 12 inclusive, for anyone who has a knowledge of elementary trigonometry. It is not necessary to know how they were derived to use them, however, as all that is needed is the ability to read a table of sines and tangents.

Formulas 5, 6, 7 and 8 are the same as for Brown & Sharpe standard gears. The dimensions at the small end of the tooth given by Formulas 10, 11 and 19 obviously are to the corresponding dimensions at the large end, as the distance from the small end of the tooth to the vertex of the pitch cone is to the pitch cone radius. This relation is expressed by these formulas. The derivation of Formula 20 may be understood by reference to Fig. 12:

CHART FOR SOLUTION OF BEVEL GEAR PROBLEMS.—III

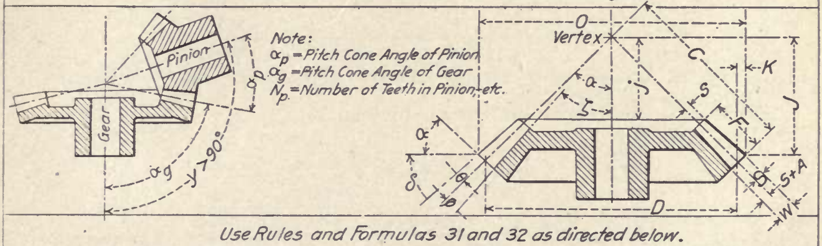
Bevel Gears with Shafts at an Acute Angle.



Use Rules and Formulas 28-30, and 4-21 in the order given.

No.	To Find	Rule	Formula
28	Pitch Cone Angle (or Edge Angle) of Pinion	Divide the sine of the center angle by the sum of the cosine of the center angle and the quotient of number of teeth in the gear divided by the number of teeth in the pinion; this gives the tangent	$\operatorname{Tan} \alpha_p = \dfrac{\operatorname{Sin} y}{\dfrac{N_g}{N_p} + \operatorname{Cos} y}$
29	Pitch Cone Angle (or Edge Angle) of Gear	Divide the sine of the center angle by the sum of the cosine of the center angle and the quotient of the number of teeth in the pinion divided by the number of teeth in the gear; this gives the tangent	$\operatorname{Tan} \alpha_g = \dfrac{\operatorname{Sin} y}{\dfrac{N_p}{N_g} + \operatorname{Cos} y}$
30	Proof of Calculations for Pitch Cone Angles	The sum of the pitch cone angles of the pinion and gear equals the center angle	$\alpha_p + \alpha_g = y$

Bevel Gears with Shafts at an Obtuse Angle.



Use Rules and Formulas 31 and 32 as directed below.

No.	To Find	Rule	Formula
31	Pitch Cone Angle (or Edge Angle) of Pinion	Divide the sine of 180 degrees minus the center angle by the difference between the quotient of the number of teeth in the gear divided by the number of teeth in the pinion and the cosine of 180 degrees minus the center angle; this gives the tangent	$\operatorname{Tan} \alpha_p = \dfrac{\operatorname{Sin}(180° - y)}{\dfrac{N_g}{N_p} - \operatorname{Cos}(180° - y)}$
32	Whether Gear is a Regular Bevel Gear, a Crown Gear, or an Internal Bevel Gear	Add 90 degrees to the pitch cone angle of the pinion. If the sum is greater than the center angle use rules and formulas 33, 30 and 4-21 in the order given. If the sum equals the center angle see rules and formulas for crown gear. If the sum is less than the center angle see rules and formulas for internal bevel gear.	
33	Pitch Cone Angle (or Edge Angle) of Gear	Divide the sine of 180 degrees minus the center angle by the difference between the quotient of the number of teeth in the pinion divided by the number of teeth in the gear and the cosine of 180 degrees minus the center angle; this gives the tangent	$\operatorname{Tan} \alpha_g = \dfrac{\operatorname{Sin}(180° - y)}{\dfrac{N_p}{N_g} - \operatorname{Cos}(180° - y)}$

$$D' = \frac{D}{\cos\alpha} = \frac{N}{P \times \cos\alpha}, \text{ also } D' = \frac{N'}{P}$$

$$\text{therefore } \frac{N'}{P} = \frac{N}{P \times \cos\alpha}, \text{ or } N' = \frac{N}{\cos\alpha}.$$

Formula 21 for checking the calculations will also be understood from Fig. 12, where it will be seen that

$$O = 2\,a\,b \times \cos\delta, \text{ also that } a\,b = \frac{C}{\cos\theta},$$

$$\text{therefore } O = \frac{2\,C \times \cos\delta}{\cos\theta}.$$

Formulas 22 to 27 inclusive are simply the corresponding Formulas 1, 9, 14, 15, 16 and 20 when $\alpha = 45$ degrees.

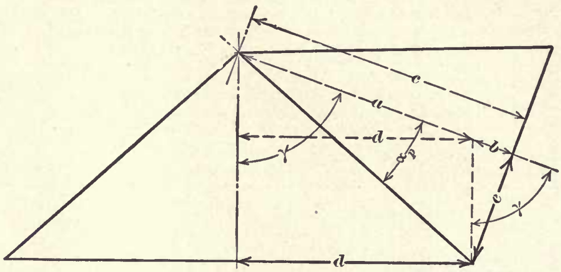


Fig. 13. Diagram for Obtaining Pitch Cone Angle of Acute Angle Gearing

Formula 28 is derived as shown in Fig. 13.

$$c = \frac{e}{\tan\alpha_p}, \text{ also, } c = a + b = \frac{d}{\sin\gamma} + \frac{e}{\tan\gamma},$$

$$\text{therefore, } \frac{e}{\tan\alpha_p} = \frac{d}{\sin\gamma} + \frac{e}{\tan\gamma}.$$

Solving for $\tan\alpha_p$, we have: $\tan\alpha_p = \dfrac{e\,(\sin\gamma \times \tan\gamma)}{d\tan\gamma + e\sin\gamma}$.

Dividing both numerator and denominator by $e\tan\gamma$, we have:

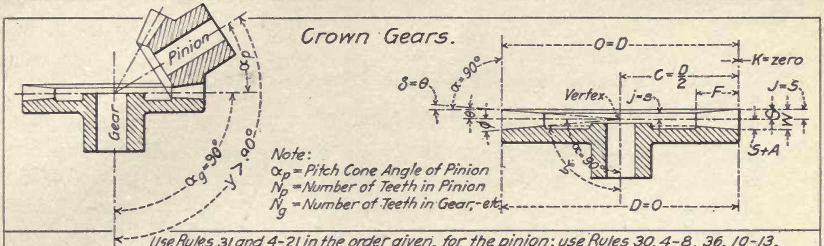
$$\tan\alpha_p = \frac{\sin\gamma}{\dfrac{d}{e} + \dfrac{\sin\gamma}{\tan\gamma}}$$

Since $d = \dfrac{N_g}{2\,P}$ and $e = \dfrac{N_p}{2\,P}$, and since $\dfrac{\sin}{\tan} = \cos$, we have:

$$\tan\alpha = \frac{\sin\gamma}{\dfrac{N_g}{N_p} + \cos\gamma}$$

CHART FOR SOLUTION OF BEVEL GEAR PROBLEMS.—IV

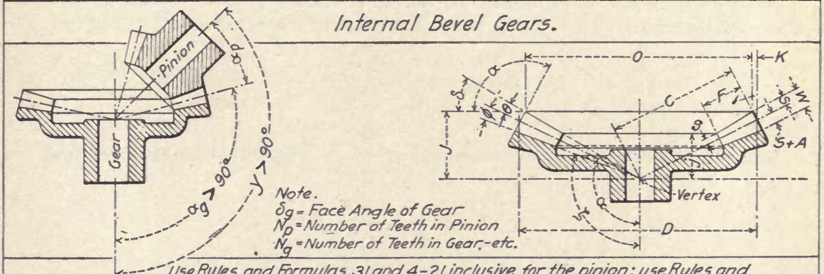
Crown Gears.



Use Rules 31 and 4-21 in the order given, for the pinion; use Rules 30, 4-8, 36, 10-13, 37, 15 and 38 in the order given for the crown gear; if dimensions for crown gear are known, to find center angle and dimensions of pinion, use rules and formulas 34, 35 and 4-21 in the order given

No.	To Find	Rule	Formula
34	Pitch Cone Angle (or Edge Angle) of Pinion	Divide the number of teeth in the pinion by the number of teeth in the gear, to get the sine	$\sin \alpha_p = \frac{N_p}{N_g}$
35	Center Angle	Add 90 degrees to the pitch cone angle of the pinion	$y = 90° + \alpha_p$
36	Pitch Cone Radius	Divide the pitch diameter by 2	$C = \frac{D}{2}$
37	Face Angle of Gear	The face cone angle of the gear equals the addendum angle	$\delta_g = \theta$
38	Number of Teeth in Equivalent Spur Gear	The teeth are equivalent in form to rack teeth	N'_g = infinity

Internal Bevel Gears.



Use Rules and Formulas 31 and 4-21 inclusive for the pinion; use Rules and Formulas 39, 30, 40, 41, 15, 42, 43, 18, 19, 44 and 21 in the order given for the gear

No.	To Find.	Rule	Formula
39	Pitch Cone Angle (or Edge Angle) of Gear	Divide the sine of 180 degrees minus the center angle, by the difference between the cosine of 180 degrees minus the center angle and the quotient of the number of teeth in the pinion divided by the number of teeth in the gear; subtract the angle whose tangent is thus found from 180 degrees	$\tan \alpha_a = \frac{\sin(180-y)}{\cos(180-y) - \frac{N_p}{N_g}}$ $\alpha_g = 180 - \alpha_a$
40	Pitch Cone Radius	Divide the pitch diameter by twice the sine of 180 degrees minus the pitch cone angle	$C = \frac{D_g}{2 \sin(180-\alpha_g)}$
41	Face Angle of Gear	Subtract 90 degrees from the sum of the pitch cone angle and the addendum angle	$\delta_g = \alpha_g + \theta - 90°$
42	Angular Addendum of Gear	Multiply the addendum by the cosine of 180 degrees minus the pitch cone angle	$K_g = S \times \cos(180-\alpha_g)$
43	Outside (or Edge) Diameter of Gear	Subtract twice the angular addendum from the pitch diameter	$O_g = D_g - 2K_g$
44	Number of Teeth in Equivalent Internal Spur Gear	Divide the number of teeth by the cosine of 180 degrees minus the pitch cone angle	$N'_g = \frac{N_g}{\cos(180-\alpha_g)}$

Formula 29 is derived by the same process for the other gear. Formula 31 (and likewise 33) is derived from Fig. 14, using the following fundamental equation:

$$\frac{e}{\tan a_p} = \frac{d}{\sin (180° - \gamma)} - \frac{e}{\tan (180° - \gamma)}$$

When solved for $\tan a_p$, this gives Formula 31.

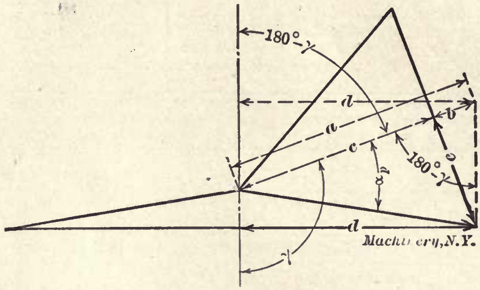


Fig. 14. Diagram for Obtaining Pitch Cone Angle of Obtuse Angle Gearing

Rule 32, of course, simply expresses the operation of finding out whether the pitch cone angle of the gear is less, equal to or greater than 90 degrees. The derivation of Formula 34 is shown in Fig. 15:

$$\sin a_p = \frac{e}{d} = \frac{N_p}{N_g}$$

Since in a crown gear the dimension $\frac{D'}{2}$ in Fig. 12 is to be measured

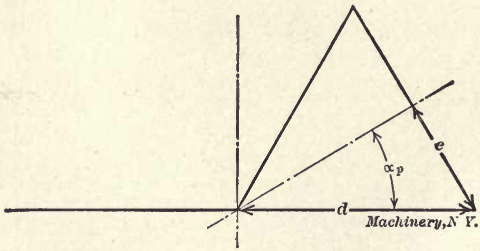


Fig. 15. Diagram for Obtaining Pitch Cone Angle of Pinion to Mesh with Crown Gear

parallel to the axis, and will therefore be of infinite length, the form of the teeth will correspond to those of a spur gear having a radius of infinite length, that is to say, to a rack. This accounts for Formula 38.

Formulas 39, 40, 42 and 44 are simply the corresponding Formulas 33, 9, 16 and 20 changed to avoid the use of negative cosines, etc., which occur with angles greater than 90 degrees. These negative functions might possibly confuse readers whose knowledge of trigonometry is elementary. The other formulas for internal gears are readily comprehensible from an inspection of Fig. 11.

CHAPTER II

EXAMPLES OF BEVEL GEAR CALCULATIONS

A number of examples of calculations are here given for practice, covering all the various types shown in Figs. 3 to 8 inclusive. The conditions of the various examples differ from each other only in the center angle. While such great accuracy is not required in the work itself, it will be found convenient in the calculations to use tables of sines and tangents which give readings for minutes to five figures. This permits accurate checking of the various dimensions by Rules and Formulas 3, 21, etc.

Shafts at Right Angles

Let it be required to make the necessary calculations for a pair of bevel gears in which the shafts are at right angles; diametral pitch = 3, number of teeth in gear = 60, number of teeth in pinion = 15, and width of face = 4 inches.

$$\tan a_p = 15 \div 60 = 0.25000 = \tan 14° \ 2' \qquad (1)$$

$$\tan a_g = 60 \div 15 = 4.00000 = \tan 75° \ 58' \qquad (2)$$

$$\gamma = 14° \ 2' + 75° \ 58' = 90° \qquad (3)$$

$$D_p = 15 \div 3 = 5.000'' \qquad (4)$$

$$S = 1 \div 3 = 0.3333'' \qquad (5)$$

$$S + A = \frac{1.157}{3} = 0.3856'' \qquad (6)$$

$$W = \frac{2.157}{3} = 0.7190'' \qquad (7)$$

$$T = \frac{1.571}{3} = 0.5236'' \qquad (8)$$

$$C = \frac{5}{2 \times 0.24249} = 10.3097'' \qquad (9)$$

$$s = 0.3333 \times \frac{6.31}{10.31} = 0.2040'' \qquad (10)$$

$$t = 0.5236 \times \frac{6.31}{10.31} = 0.3204'' \qquad (11)$$

$$\tan \theta = \frac{0.3333}{10.3097} = 0.03233 = \tan 1° \ 51' \qquad (12)$$

$$\tan \phi = \frac{0.3856}{10.3097} = 0.03740 = \tan 2° \ 9' \qquad (13)$$

$$\delta = 90° - (14° \ 2' + 1° \ 51') = 74° \ 7' \qquad (14)$$

$$\zeta = 14° \ 2' - 2° \ 9' = 11° \ 53' \qquad (15)$$

$$K = 0.3333 \times 0.97015 = 0.3234'' \qquad (16)$$

$$O = 5.000 + 2 \times 0.3234 = 5.6468'' \qquad (17)$$

$$J = \frac{5.6468}{2} \times 3.51441 = 9.9225'' \qquad (18)$$

$$j = 9.9225 \times \frac{6.31}{10.31} = 6.0726'' \qquad (19)$$

$$N' = \frac{15}{0.97015} = 15.4 \qquad (20)$$

$$5.6468'' \cong \frac{20.6194 \times 0.27368}{0.99948} = 5.6461'' \qquad (21)$$

This gives all the data required for the pinion. Rules 5 to 13 inclusive apply equally to the gear and the pinion, so we have only calculations by Rules and Formulas 4 and 14 to 21 to make, though it is well to calculate Formula 9 a second time as a check for the same calculation for the pinion.

$$D = \frac{60}{3} = 20.000'' \qquad (4)$$

$$C = \frac{20}{2 \times 0.97015} = 10.3077'' \qquad (9)$$

$$\delta = 90 - (75^\circ\ 58' + 1^\circ\ 51') = 12^\circ\ 11' \qquad (14)$$

$$\zeta = 75^\circ\ 58' - 2^\circ\ 9' = 73^\circ\ 49' \qquad (15)$$

$$K = 0.3333 \times 0.24249 = 0.0808'' \qquad (16)$$

$$O = 20 + 2 \times 0.0808 = 20.1616'' \qquad (17)$$

$$J = \frac{20.1616}{2} \times 0.2159 = 2.1764'' \qquad (18)$$

$$j = 2.1764 \times \frac{6.31}{10.31} = 1.3320'' \qquad (19)$$

$$N' = \frac{60}{0.24249} = 247 \qquad (20)$$

$$20.1616'' \cong \frac{20.6154 \times 0.97748}{0.99948} = 20.1615'' \qquad (21)$$

This gives the calculations necessary for this pair of gears, which are shown drawn and dimensioned in Fig. 19. There are two or three other dimensions, such as the over-all length of the pinion, etc., which depend on arbitrary dimensions given the gear blank. Directions for calculating these are given in the text in connection with Fig. 19.

Acute Angle Bevel Gearing

Let it next be required to calculate the dimensions of a pair of bevel gears whose center angle is 75 degrees, the number of teeth in the pinion 15, the number of teeth in the gear 60, the diametral pitch 3, and the width of face 4 inches. This is the same as the first example, except for the center angle. Following the directions given in the chart we have:

$$\tan \alpha_p = \frac{0.96593}{\frac{60}{15} + 0.25882} = 0.22681 = \tan 12^\circ\ 47' \ldots\ldots\ldots\ldots\ldots (28)$$

$$\tan \alpha_g = \frac{0.96593}{\frac{15}{60} + 0.25882} = 1.89837 = \tan 62^\circ\ 13' \ldots\ldots\ldots\ldots\ldots (29)$$

$$\gamma = 12^\circ\ 47' + 62^\circ\ 13' = 75^\circ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

Formulas 4 to 8 as in first example; also, $C = 11.2989''$, $s = 0.2154''$, $t = 0.3382''$, $\theta = 1^\circ\ 41'$, $\phi = 1^\circ\ 57'$, $\delta = 75^\circ\ 32'$, $\zeta = 10^\circ\ 50'$, $K = 0.3251''$, $O = 5.6502''$, $J = 10.9501''$, $j = 7.0748''$, and $N' = 15.3$, also,

$$5.6502'' \cong \frac{22.598 \times 0.24982}{0.99957} = 5.6483'' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

For the gear, the additional calculations give: $C = 11.303''$, $\delta = 26^\circ\ 6'$, $\zeta = 60^\circ\ 16'$, $K = 0.1553''$, $O = 20.3106''$, $J = 4.9748''$, $j = 3.2142''$, $N' = 129$.

$$20.3106'' \cong \frac{22.606 \times 0.89803}{0.99957} = 20.3096'' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

The above calculations are not all given in full, as most of them are merely re-duplications of formulas previously used.

Crown Gear

Suppose it is required to make a crown gear and a pinion for the same number of teeth, pitch and face as in the previous example. What are the additional calculations necessary? Following the proper formulas in the order given by the chart, we have:

$$\sin \alpha_p = \frac{15}{60} = 0.25000 = \sin\ 14^\circ\ 29' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

$$\gamma = 90^\circ + 14^\circ\ 29' = 104^\circ\ 29' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

The other calculations are similar to those already given.

Internal Bevel Gear

Let it be required to design a pair of bevel gears of the same number of teeth, pitch and face, in which the center angle is 115 degrees. This being an example of obtuse angle gearing, we use Formula 31.

$$\tan \alpha_p = \frac{0.90631}{\frac{60}{15} - 0.42262} = 0.25334 = \tan\ 14^\circ\ 13' \ldots\ldots\ldots\ldots\ldots (31)$$

Thus, according to Rule 32, we find that

$$14^\circ\ 13' + 90^\circ = 104^\circ\ 13' \quad 115^\circ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

showing that the gear is an internal bevel gear. Applying the rules and formulas for internal bevel gearing, we have:

$$\tan a_a = \frac{0.90631}{0.42262 - \frac{15}{60}} = 5.25032 = \tan 79^\circ\ 13'$$

$$180^\circ - 79^\circ\ 13' = 100^\circ\ 47' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

$$\gamma = 100^\circ\ 47' + 14^\circ\ 13' = 115^\circ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

$$C = \frac{20}{2 \times 0.98234} = 10.1797'' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

$$\delta = 100^\circ\ 47' + 1^\circ\ 53' - 90^\circ = 12^\circ\ 40' \ldots\ldots\ldots\ldots\ldots (41)$$

$$\zeta = 98^\circ\ 37', \text{ and } K = 0.0624''$$

$$O = 20 - 2 \times 0.0624 = 19.8752'' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

$$N' = \frac{60}{0.1871} = 320 \text{ (internal)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

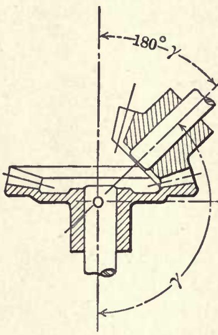


Fig. 16. Internal Bevel Gearing

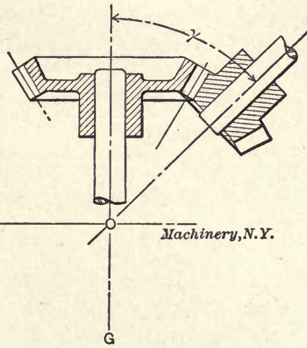


Fig. 17. Acute Bevel Gearing Used as Substitute for Internal Gearing in Fig. 16

The calculations for the pinion and the other calculations for the gear are similar to those already given.

Obtuse Angle Bevel Gearing

Let it be required to calculate the dimensions of the same set of gears but with the center angle of 100 degrees. This being an example of obtuse angle gearing, we apply Formula 31 as follows:

$$\tan a_p = \frac{0.98481}{\frac{60}{15} - 0.17365} = 0.25738 = \tan 14^\circ\ 26' \ldots\ldots\ldots\ldots (31)$$

and thus discover that it is an example of reguiar obtuse angle gearing, since

$$14^\circ\ 26' + 90^\circ = 104^\circ\ 26' > 100^\circ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

The remaining calculations for the angles are as follows:

$$\tan \alpha = \frac{0.98481}{\frac{15}{60} - 0.17365} = 12.8986 = \tan 85° \; 34' \ldots\ldots\ldots\ldots\ldots (33)$$

$$\gamma = 14° \; 26' + 85° \; 34' = 100° \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

and the calculations for the other dimensions as per the table.

How to Avoid Internal Bevel Gears

When Rule 32, in any given case, shows that the large gear will be an internal bevel gear, such as shown in Fig. 16, this construction may be avoided without changing the position of the shafts, the numbers of the teeth in the gear, the pitch of the teeth, or the width of face. This is done simply by subtracting the given center angle from 180 degrees, and using the remainder as a new center angle in calculating a set of acute angle gears by Rules and Formulas 28, 29, 30, etc. A pair of bevel gears calculated on this basis corresponding to those in Fig. 16 is shown in Fig. 17. It will be seen that the contact takes place on the other side of the axis *OP* of the pinion.

It is necessary to avoid internal bevel gears as it is practically impossible to cut them. It may be that some forms of templet planing machines will do this work, if the pitch cone angle is not too great, but no form of generating machine will do it. It is rather doubtful if any one has ever cut a pair of internal bevel gears, though the writer has seen occasional examples of cast gears of this type.

CHAPTER III

SYSTEMS OF TOOTH OUTLINES USED FOR BEVEL GEARING

Five systems of tooth outlines are commonly used for bevel gearing. They are the cycloid, the standard 14½-degree involute, the 20-degree involute and the 15- and 20-degree octoid.

The Cycloidal System

The cycloidal form of tooth is obsolete for cut bevel gears, and is rarely met with nowadays for cast gears even. It requires very careful workmanship, and is difficult or impossible to generate. It is also a bad shape to form with a relieved cutter, as the cutting edge tends to drag at the pitch line, where for a short distance the sides of the teeth are nearly or quite parallel. For spur gearing it has a few points of advantage over the involute form of tooth, but in the case of bevel gearing these are nullified by the impossibility of generating the teeth in practicable machines. The cycloidal form of tooth need not be seriously considered for bevel gears.

Involute and Octoid Teeth

Most bevel gears are made on the involute system, of either the standard 14½-degree pressure angle, or the 20-degree pressure angle. In spur gear teeth the pressure angle may be defined as the angle which the flat surface of the rack tooth makes with the perpendicular to the pitch line. The 20-degree tooth is consequently broader at the base and stronger in form than the 14½-degree tooth. This same difference applies to bevel gears. Most bevel gears that are milled with formed cutters are made to the 14½-degree standard, as cutters for this shape are regularly carried in stock. The planed gears, made by the templet or generating principles, are nowadays often made to the 20-degree pressure angle, both for the sake of obtaining stronger teeth, and for avoiding undercutting of the flanks of the pinions as well. This undercutting is due to the phenomenon of "interference," as it is called, which is minimized by increasing the pressure angle.

If you ask the manufacturer to plane a pair of involute bevel gears for you on the Bilgram, Gleason or other similar generating machine, he will not give you involute teeth, but something "just as good." This "just as good" form was invented by Mr. Bilgram, and was named "Octoid" by Mr. Geo. Grant. In generating machines the teeth of the gears are shaped by a tool which represents the side of the tooth of an imaginary crown gear. The cutting edge of the tool is a straight line, since the imaginary crown gear has teeth whose sides are plane surfaces. It can be shown that the teeth of a true involute crown gear have sides which are very slightly curved. The minute difference between the tooth shapes produced by a plane crown tooth and a slightly curved crown tooth is the minute difference between the octoid and involute forms. Both give theoretically correct action. The customer in ordering gears never uses the word "octoid," as it is not a commercial term; he calls for "involute" gears.

Formed Cutters for Involute Teeth

For 14½-degree involute teeth, the shapes of the standard cutter series furnished by the makers of formed gear cutters are commonly used. There are 8 cutters in the series, to cover the full range from the 12-tooth pinion to a crown gear. The various cutters are numbered from 1 to 8, as given in the table below:

No. 1 will cut wheels from 135 teeth to a rack.
No. 2 will cut wheels from 55 teeth to 134 teeth.
No. 3 will cut wheels from 35 teeth to 54 teeth.
No. 4 will cut wheels from 26 teeth to 34 teeth.
No. 5 will cut wheels from 21 teeth to 25 teeth.
No. 6 will cut wheels from 17 teeth to 20 teeth.
No. 7 will cut wheels from 14 teeth to 16 teeth.
No. 8 will cut wheels from 12 teeth to 13 teeth.

It should be remembered that the number of teeth in this table refers to the number of teeth in the equivalent spur gear, as given by Rule 20, which should always be used in selecting the cutter used for milling the teeth of bevel gears. Thus for the gear in the first example in Chapter II, the No. 1 cutter should be used. The standard bevel gear cutter is made thinner than the standard spur gear cutter, as it must pass through the narrow tooth space at the inner end of the face. As usually kept in stock, these cutters are thin enough for bevel gears in which the width of face is not more than one-third the pitch cone radius. Where the width of face is greater, special cutters have to be made, and the manufacturer should be informed as to the thickness of the tooth space at the small end; this will enable him to make the cutter of the proper width.

Special Forms of Bevel Gear Teeth

In generating machines (such as the Bilgram and the Gleason) it is often advisable to depart from the standard dimensions of gear teeth as given by Rules and Formulas 1 to 44. For instance, where the pinion is made of bronze and the gear of steel, the teeth of the former can be made wider and those of the latter correspondingly thinner, so as to somewhere nearly equalize the strength of the two. Again, where the pinion has few teeth and the gear many, it may be advisable to make the addendum on the pinion larger and the dedendum correspondingly smaller, reversing this on the gear, making the addendum smaller and the dedendum larger. This is done to avoid interference and consequent undercut on the flanks of pinions having a small number of teeth. Such changes are easily effected on generating machines and instructions for doing this for any case will be furnished by the makers.

CHAPTER IV

STRENGTH AND DURABILITY OF BEVEL GEARS

The same materials are used in general for making bevel gears as for spur gears and each has practically the same advantages and disadvantages for both cases. In general, the strength of different materials is roughly proportional to the durability.

The Materials Used for Making Bevel Gears

Cast iron is used for the largest work, and for smaller work which is not to be subjected to heavy duty. In cases where great working stress or a sudden shock is liable to come on the teeth, steel is ordinarily used. Such gears are made from bar stock for the smallest work, from drop forgings for intermediate sizes made on a manufacturing basis, and from steel castings for heavy work. The softer grades of steel are not fitted for high-speed service, as this material abrades more rapidly than cast iron. This objection does not apply to hardened steels, such as used in automobile transmission gears.

As in the case of spur gearing it is quite common to make the gear and pinion of different materials. This is advantageous from the standpoint of both efficiency and durability, since two dissimilar metals work on each other with less friction than similar metals, as is well known. Cast iron and steel, and steel and bronze are common combinations. In general, the pinion should be made of the stronger material, since it is of weak form; and it should be made of the more durable material, as it revolves more rapidly and each tooth comes into working contact more times per minute than do those of the larger mating gear. In a steel and cast iron combination, then, the pinion should be of steel, while the gear is of cast iron. In a steel and bronze combination, the pinion should be of steel and the gear of bronze, though this is more costly than when the materials are reversed.

A wide range of physical qualities is now available in steel, both for parts small enough to be made from bar stock, and for those made from drop forgings. Recent improvements have also given almost as much flexibility in the choice of steel castings. Gears made from high grade steels may be subjected to heat treatments which increase their durability and strength amazingly.

Raw-hide and fiber are quite largely used for pinion blanks in cases where it is desired to run gearing at a very high speed and with as little noise as possible. There is a little more difficulty in building up a raw-hide blank properly for a bevel gear than for a spur gear. Fiber, which is used in somewhat the same way, has the merit of convenience and comparative inexpensiveness, as it may be purchased in a variety of sizes of bars, rods, tubes, etc., ready to be worked up into pinion blanks at short notice. It is not so strong as raw-hide,

STRENGTH OF BEVEL GEARS

List of Reference Letters.

D = pitch diameter of gear in inches.

R = revolutions per minute.

V = velocity in ft. per min. at pitch diameter.

S_s = allowable static unit stress for material.

S = allowable unit stress for material at given velocity.

F = width of face.

N′ = No. of teeth in equivalent spur gear (See diagram).

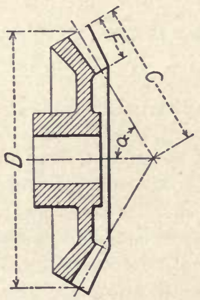
Y = outline factor (see table below).

P = diametral pitch (if circular pitch is given, divide 3.1416 by circular pitch to obtain diametral pitch).

C = pitch cone radius.

W = maximum safe tangential load in pounds at pitch diameter.

H.P. = maximum safe horse power.



$$N' = \frac{\text{Number of teeth}}{\cos \alpha}$$

(Rule No. 20)

Table of Outline Factors (Y) for $14\frac{1}{2}°$ and 20° Involute

N′	Outline Factor = Y: $14\frac{1}{2}°$ Involute (Std.)	Outline Factor = Y: 20° Involute	N′	Outline Factor = Y: $14\frac{1}{2}°$ Involute (Std.)	Outline Factor = Y: 20° Involute
12	0.210	0.245	27	0.314	0.349
13	0.220	0.261	30	0.320	0.358
14	0.226	0.276	34	0.327	0.371
15	0.236	0.289	38	0.336	0.383
16	0.242	0.295	43	0.346	0.396
17	0.251	0.302	50	0.352	0.408
18	0.261	0.308	60	0.358	0.421
19	0.273	0.314	75	0.364	0.434
20	0.283	0.320	100	0.371	0.446
21	0.289	0.327	150	0.377	0.459
23	0.295	0.333	300	0.383	0.471
25	0.305	0.339	Rack	0.390	0.484

Use rules and formulas 45–48 in the order given

No.	To Find	Rule	Formula
45	Velocity in ft. per min. at the pitch diameter	Multiply the product of the diameter in inches and the number of revolutions per minute, by 0.262	$V = 0.262\ DR$
46	Allowable unit stress at given velocity	Multiply the allowable static stress by 600 and divide the result by the velocity in feet per minute plus 600	$S = S_s \times \frac{600}{600+V}$
47	Maximum safe tangential load at pitch diameter	Multiply together the allowable stress for the given velocity, the width of face, the tooth outline factor and the difference between the pitch cone radius and the width of face; divide the result by the product of the diametral pitch and the pitch cone radius	$W = \frac{SFY(C-F)}{PC}$
48	Maximum safe Horse Power	Multiply the safe load at the pitch line by the velocity in feet per minute, and divide the result by 33,000	$HP = \frac{WV}{33{,}000}$

and is difficult to machine owing to its gritty composition. For light duty at high speed it does very well. For large, high-speed gearing it was formerly a common practice to use inserted wooden teeth on the gear, meshing with a solid cast iron pinion. This construction is seldom used for cut gearing.

Strength of Bevel Gear Teeth

The Lewis formula is the one generally used in this country for calculating the strength of gears. Mr. Myers, who had an article on the "Strength of Gears" in the December, 1906, issue of MACHINERY, gives Mr. Barth's adaptation of this formula for calculating the strength of bevel gears. The rules and formulas on the next page are condensed from the method given in the article referred to.

The factors to be taken into account are the pitch diameter of the gear, the number of revolutions per minute, the diametral pitch (or circular pitch as the case may be) the width of face, the pitch cone radius, the number of teeth in the gear and the maximum allowable static fiber stress for the material used. From this we may find the maximum allowable load at the pitch line, and the maximum horse-power the gear should be allowed to transmit.

The reader familiar with the Lewis formula will note that Rule and Formula 47 is the same as for spur gears with the exception of the additional factor $\frac{C - F}{C}$. This factor is an approximate one which expresses the ratio of the strength of a bevel gear to that of a spur gear of the same pitch and number of teeth, the decrease being due to the fact that the pitch grows finer toward the vertex. This factor is approximate only and should not be used for cases in which F is more than 1/3 C; but since no bevel gears should be made in which F is more than 1/3 C, the rule is of universal application for good practice. As the width of face is made greater in proportion to the pitch cone radius, the increase of strength obtained thereby grows proportionately smaller and smaller, as may be easily proved by analysis and calculation. Actually the advantage of increasing the width of face is even less than is indicated by calculation, since the unavoidable deflection and disalignment of the shaft is sure at one time or another to throw practically the whole load on the weak inner ends of the teeth, which thus have to carry the load without help from the large pitch at the outer ends.

Rules and Formulas for the Strength of Bevel Gears

The reference letters, rules and formulas on the next page, for the strength of bevel gear teeth, are self-explanatory. As an approximate guide for ordinary calculations, 8,000 pounds per square inch may be allowed for the static stress of cast iron and 20,000 pounds for ordinary machine steel or steel castings. Where the gearing is to be subjected to shock, 6,000 pounds for cast iron and 15,000 pounds for steel are more satisfactory figures. The wide range of materials offered the designer, however, makes any fixed tabulation of fiber stress impracticable. An example showing the use of these rules and formulas is given herewith.

Calculate the maximum load at the pitch line which can be safely allowed for the bevel gears in Fig. 19, if the maximum allowable static stress for the pinion is 20,000 pounds, and for the gear, 8,000 pounds per square inch; the pinion runs at 300 revolutions per minute. The calculations for the pinion are as follows:

$$N' = \frac{15}{\cos 14^\circ} = 15.5, \text{ approx.}$$

$$V = 0.262 \times 5 \times 300 = 400 \text{ feet per minute (about)} \quad (45)$$

$$S = 20{,}000 \times \frac{600}{600 + 400} = 12{,}000 \text{ pounds per square inch} \quad (46)$$

$$W = \frac{12{,}000 \times 4 \times 0.292 \times 6.3}{3 \times 10.3} = 2{,}860 \text{ pounds} \quad (47)$$

For the gear, the velocity is the same as for the pinion. The necessary calculations are as follows:

$$N' = \frac{60}{\cos 76^\circ} = 250, \text{ approx.}$$

$$S = 8{,}000 \times \frac{600}{600 + 400} = 4{,}800 \text{ pounds per square inch} \quad (46)$$

$$W = \frac{4{,}800 \times 4 \times 0.467 \times 6.3}{3 \times 10.3} = 1{,}830 \text{ pounds} \quad (47)$$

The gear is, therefore, the weaker of the two, and thus limits the allowable tooth pressure. The maximum horse-power this gearing will transmit safely is found as follows:

$$\text{H. P.} = \frac{1{,}830 \times 400}{33{,}000} = 22 \quad (48)$$

Durability is practically of as much importance as strength in proportioning bevel gears, but unfortunately no data are as yet available for making satisfactory comparisons of durability, so that the usual procedure is to design the gears for strength alone, assuming then that they will not wear out within the lifetime of the machine in which they are used.

CHAPTER V

DESIGN OF BEVEL GEARING

So far we have dealt with design as relating to calculations. In this chapter will be discussed the application of the calculated dimensions, the determination of the factors left to the judgment, and the recording of the design in the drawing.

Bevel Gear Blanks

Various forms may be given to the blanks or wheels on which bevel gear teeth are cut, depending on the size, material, service, etc., to be provided for. The pinion type of blank is shown in Fig. 12 and elsewhere. It is used mostly, as indicated by the name, for gears of a small number of teeth and small pitch cone angle. Where the diameter of the bore comes too near to the bottoms of the teeth at the

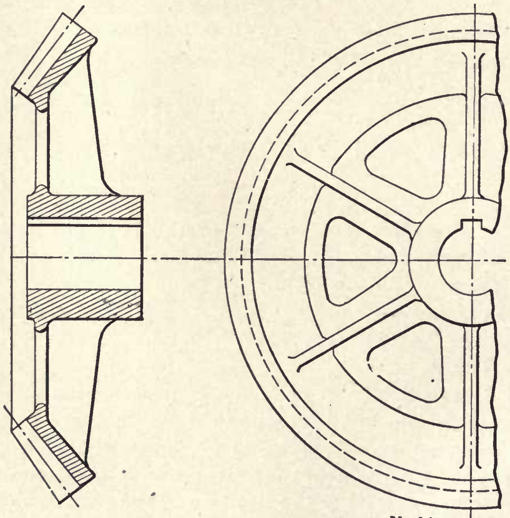


Fig. 18. T-arm Style of Bevel Wheel for Heavy Work

small end, it is customary to omit the recess indicated by dimension z, and leave the front face of the pinion blank as in the case shown in Fig. 19.

For gears of a larger number of teeth, the web type shown in Fig. 9 and elsewhere is appropriate. This does not require to be finished all over, as the sides of the web, the outside diameter of the hub, and the under side of the rim may be left rough if desired.

A steel gear suitable for very heavy work is shown in Fig. 18. Here the web is reinforced by ribs. The web may be cut out so that the rim is supported by T-shaped arms, as shown. This makes a very stiff wheel and at the same time a very light one, when its strength is considered. Where the pitch cone angle is so great that the strengthening rib would be rather narrow at the flange, it may be given the form shown in Fig. 19 in place of that shown in Fig. 18.

General Considerations Relating to Design

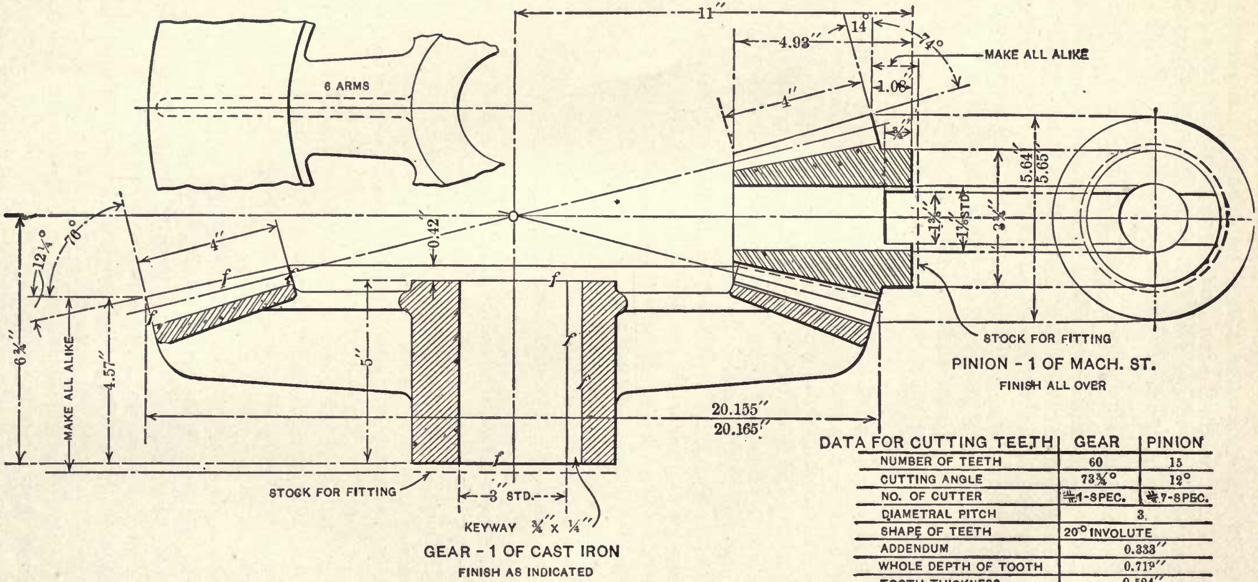
The performance of the most carefully designed and made bevel gears depends to a considerable extent on the design of the machine in which they are used. When the shafts on which a pair of bevel gears are mounted are poorly supported or poorly fitted in their bearings, the pressure of the driving gear on the driven, causes it to climb up on the latter, throwing the shafts out of alignment. This in turn causes the teeth to bear with a greater pressure at one end of the face (usually on the outer end) than the other, thus making the tooth more liable to break than is the case where the pressure is more evenly distributed. It is important, therefore, to provide rigid shafts and bearings and careful workmanship for bevel gearing.

The question of alignment of the shafts should be considered in deciding on the width of face of the gear. Making the width of the face more than one-third of the pitch cone radius adds practically nothing to the strength of the gear even theoretically, since the added portion is progressively weaker as the tooth is lengthened, as has been explained. In addition to this, there is the danger that through springing of the shafts or poor workmanship, the load will be thrown onto the weak end of the tooth, thus fracturing it. For this reason it may be laid down as a definite rule that there is nothing to be gained by making the face of the bevel gear more than one-third of the pitch cone radius, as required by Rules 45 to 48.

The Brown & Sharpe gear book gives a rule for the maximum width of face allowable for a given pitch. The width of face should not exceed five times the circular pitch or 16 divided by the diametral pitch. This rule is also rational since the danger to the teeth from the misalignment of the shaft increases both with the width of face and with the decrease of the size of the tooth, so that both of these should be reckoned with. In designing gearing it is well to check the width of face from the rule relating to the pitch cone radius and that relating to the pitch as well, to see that it does not exceed the maximum allowed by either.

Model Bevel Gear Drawing

It is not enough for the designer to carefully calculate the dimensions of a set of bevel gearing. In addition to this he has the important task of recording these dimensions in such a form that they will be intelligible to an intelligent workman, and will plainly furnish him every point of information needed for the successful completion of the work without further calculation. A drawing which practically fills these requirements is shown in Fig. 19. The arrangement of this drawing and the amount and kind of information shown on it are based on the drafting-room practice of the Brown & Sharpe



DATA FOR CUTTING TEETH	GEAR	PINION
NUMBER OF TEETH	60	15
CUTTING ANGLE	73¾°	12°
NO. OF CUTTER	#1-SPEC.	#7-SPEC.
DIAMETRAL PITCH	3	
SHAPE OF TEETH	20° INVOLUTE	
ADDENDUM	0.333"	
WHOLE DEPTH OF TOOTH	0.719"	
TOOTH THICKNESS	0.524"	
ADDENDUM AT SMALL END	0.204"	
TOOTH THICKNESS " "	0.320"	

Machinery, N.Y.

Fig. 19. Model Working Drawing for Bevel Gears to be Cut with Formed Milling Cutter

Mfg. Co., as described by Mr. Burlingame in the article "Figuring Gear Drawings" in the August, 1906, issue of MACHINERY. Some changes and additions have been made in the arrangement of the dimensioning, however, so that firm cannot be held responsible for all that appears on the engraving.

In general, the dimensions necessary for turning the blank have been given on the drawing itself, while those for cutting the teeth are given in tabular form. All the dimensions were calculated from Rules 1 to 21 inclusive and may be checked for practice by the reader. It will be noticed that limits are given for the important dimensions. This should always be done for manufacturing work which is inspected in its course through the shop. It ought to be done even when a single gear is made, as it is exceedingly difficult to properly set a gear if the workman does not work close enough. There is no sense, however, in asking him to work to thousandths of an inch on blanks like these, so he should be given some notion as to the accuracy required by limits such as shown.

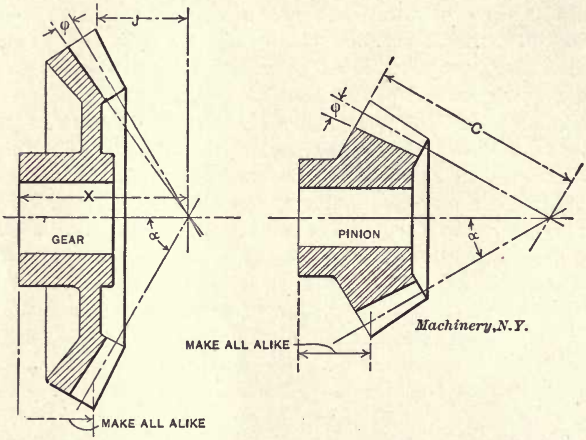
It is assumed that the gears are to be cut with rotary cutters. It is unusual to do this with a pitch as coarse as this, though there are machines on the market capable of handling such work. In gear cutting machines using form cutters, the blanks are located for axial position by the rear face of the hub. It is necessary also to leave stock at this place for fitting the gears in the machine. It will be seen that the dimension for bevel gears and pinions from the outside edge of the blank to the rear face of the hub is marked "Make all alike." This means that the same amount of stock should be left on all the gears in a given lot so that after the machine is set for one of them, it will not be necessary to alter the adjustment for the remainder.

There are one or two dimensions which are not given directly by Rules 1 to 21. One of these is the distance 4.57 inches from the outside edge of the teeth to the finished rear face of the hub of the gear. This dimension is commonly scaled from an accurate drawing, but it may be calculated by subtracting the vertex distance from the distance between the pitch cone vertex O and the rear face of the hub. This gives $6\frac{3}{4} - 2.1764$ equals 4.57 inches (about) as dimensioned. Another dimension not directly calculated is the over-all length of the pinion. This may be obtained by subtracting the vertex distance at the small end (j) from the distance between the vertex and the rear face of the hub, giving 4.93 inches as shown.

In the tabular dimensions for cutting the teeth, most of the figures are self-explanatory. The fact that in this particular case a 20-degree form of tooth has been adapted to avoid the undercut in small pinions (see Chapter III on Systems of Tooth Outlines used for Bevel Gearing) is indicated in the table.

The number of cutter is selected from the table on page 21 in accordance with the number of teeth (N') in equivalent spur gear, as determined by Rule 20. This is 15.4 for the pinion, and 247 for the gear, giving a No. 1 and No. 7 cutter respectively. These cutters are marked special, owing to the fact that they are 20 degrees involute instead of 14½ degrees. They would be special under any circumstances, however, since the width of face for these gears (4 inches) is more than 1/3 the pitch cone radius, which figures out to 10.3097 inches. Standard bevel gear cutters are only made thin enough to pass through the teeth at the small end when the width of face is not more than 1/3 the pitch cone radius. For this reason cutters thinner than the standard would have to be used.

In bevel pinions of the usual form, such as shown in Fig. 12, dimension *z* there given has to be furnished. This may be scaled from a carefully made drawing, or may be calculated by subtracting the length of the bore of the pinion from the over-all length, the latter being obtained as described for the pinion in Fig. 19. Such dimensions do



Figs. 20 and 21. Additional Dimensions for Gears to be Cut by the Templet Planing Process, or on the Gleason Generating Machine

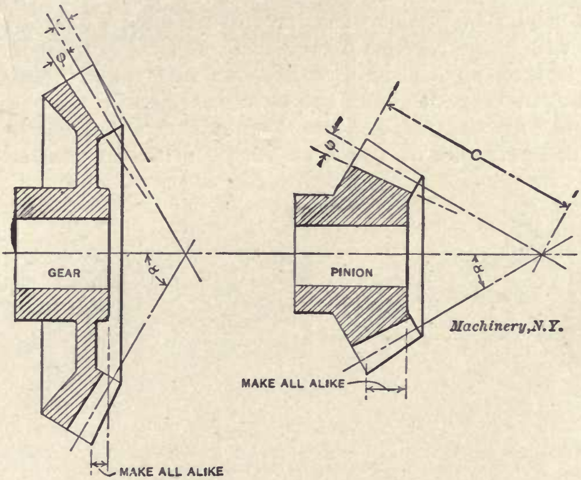
not need to be given in thousandths on moderately large work. It is also not necessary to give the angles any closer than the quarter degree, as few machines are furnished with graduations which can be read finer than this. In order to check the calculations carefully, however, it is wise, as previously described, to make them with considerable accuracy, using tables of sines and tangents which read to five figures. After the dimensions are calculated, they may be put in more approximate form for the drawing.

The gear drawing in Fig. 19 is dimensioned more fully, perhaps, than is customary, especially in shops having a large gear-cutting department, where the foreman and operators are experienced and have access to tables and records of data for bevel gear cutting. Every dimension given is useful, however, and it is a good plan to include them all, especially on large work.

Dimensioning Drawings for Gears whose Teeth are to be Planed

The machine on which the teeth of a gear are to be cut determines to some extent the dimensions which the workman needs, so this should be taken into account in making the drawing. For gears which are to be cut on a templet planing machine, the dimension given in Fig. 19 may be followed in general. Further dimensions are needed, however, to set the blank so that the vertex of the pitch cone corresponds with the central axis of the machine. For gears with pitch cone angle greater than 45 degrees, this may be obtained from dimension *X*, as given in Fig. 20, or, better, from dimension *J*. For gears smaller than 45 degrees, *C* (Fig. 21) may be given.

There are two commercial forms of gear generating machines in general use in this country for planing the teeth of bevel gears. These are the Gleason and Bilgram machines. Since the methods of supporting the gears are different, the drawings should be dimensioned to suit, if it is known beforehand how they are to be cut. For the



Figs. 22 and 23. Additional Dimensions for Gears to be Cut on the Bilgram Generating Machine

Gleason machine the dimensioning shown in Figs. 20 and 21 should be given, in addition to that shown in Fig. 19. The angles α and ϕ, the pitch cone angle and dedendum angle respectively, may well be put in the table of dimensions instead of on the drawing. The distance from the outside corner of the teeth to the rear face of the hub should be made alike for all similar gears in the lot, the same as for gears which are to be cut by the form cutters or the templet process. The cutting angle may be omitted from the drawing.

The method of dimensioning for the Bilgram gear planer is shown in Figs. 22 and 23. Angles α and ϕ should be given in the table as before. Dimension *S* is used for setting on gears of large pitch cone angle, and dimension *C* or the pitch cone radius for those of small pitch cone angle (less than 45 degrees). It is a good idea to give both of these dimensions for both gear and pinion, so that the setting may be checked by two different methods. In this machine the dimension to be marked "Make all alike" should be given as shown.

CHAPTER VI

MACHINES FOR CUTTING BEVEL GEAR TEETH

While a very large number of machines have been placed on the market, first and last, for cutting the teeth of bevel gears, the number of designs in common use in this country is small, it being possible, practically, to number them with the fingers of one hand. A brief discussion will here be given of the principles and mechanism of the more commonly used of these machines.

Spherical Basis of the Bevel Gear; Tredgold's Approximation

The principles in common use for cutting teeth of bevel gears are identical with those for cutting the teeth of spur gears, but they are modified in their application to correspond with the spherical basis of the bevel gear. Fig. 24 shows two bevel gears and a crown gear with axes *OC*, *OB*, and *OA* respectively. Fig. 25 shows their pitch surfaces, all of which converge at vertex *O*. These pitch surfaces are formed

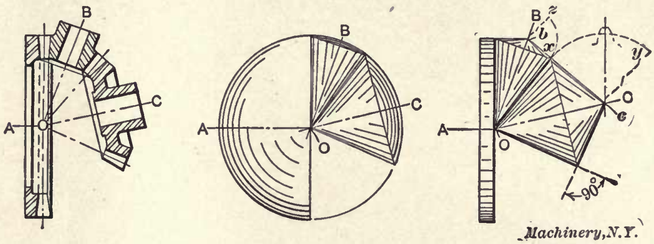


Fig. 24 Fig. 25 Fig. 26

Illustrating the Spherical Basis of Bevel Gears, and Tredgold's Approximation for Developing the Outlines of the Teeth on a Plane Surface

of cones, cut from a sphere as shown, whose center is at the vertex *O*. The pitch surface of the crown gear becomes the plane face of the hemisphere at the left of Fig. 25. To study the action of these gears the same way as we do that of spur gears when their teeth are drawn on the plane surface of the drawing board, the corresponding lines for the bevel gears would have to be drawn on the surface of the sphere from which the pitch cones were cut. The various pitch circles would be struck from centers located at the points where the axes *OA*, *OB*, and *OC* break with the surface of the sphere. The method of drawing would be identical with that for spur gears. It should be noted that straight lines, on spherical surfaces, are represented by great circles—that is to say, by the intersection of the surface with planes passing through the center of the sphere.

Owing to the impracticability of the sphere as a drawing board, a process, known as "Tredgold's Approximation," is usually followed for laying out the teeth of bevel gears. This is shown in Fig. 26 applied to the same case as in the two preceding figures. The teeth are drawn and the action studied on surfaces of cones complementary to the pitch cones—that is, on the cones with vertices at *c* and *b*. The surfaces of these cones can be developed on a flat piece of paper, as shown on axes *OB* and *OC*. In these cases the pitch line becomes *xy* and *xz*, as there illustrated. Teeth drawn on this pitch line as for a spur gear may be laid out on the conical surface and used as the outlines of bevel gear teeth. Teeth so drawn are identical with those of the equivalent spur gear illustrated in Fig. 12, as will be seen when comparing it with Fig. 26. For the crown gear, rack teeth are wrapped around the surface of the cylinder.

Principles of Action of Bevel Gear Cutting Machinery

There are three principles of action commonly used for cutting the teeth of bevel gears, namely, the form tool, the templet and the molding-generating principles. There are two machines used to some ex-

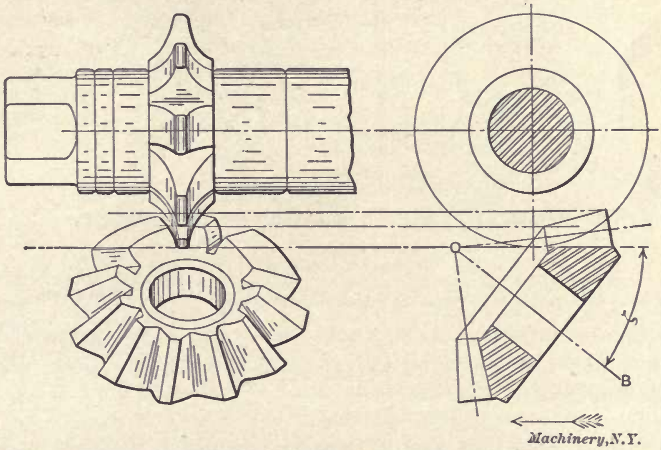


Fig. 27. Shaping the Teeth of a Bevel Gear by the Formed Cutter Process

tent in Europe which employ a fourth, that known as the odontographic principle. It is not in use in this country, so it will not be described here.

The *formed tool principle* is illustrated in Fig. 27, where a form cutter is shown shaping one side of the tooth of a bevel gear. The gear blank is tipped up to cutting angle ζ and fed beneath the cutter in the direction of the arrow. It will be immediately seen from an examination of the figure that the form tool process is by necessity approximate. It is evident that the right-hand side of the cutter is reproducing its own unchanging outline along the whole length of the face of the tool at the right. This form should not be unchanging for, as has been explained, the teeth and the space between them grow smaller towards the apex of the pitch cone, where they finally vanish, so it is evident that the outline of a tooth at the small end should be the same as that at the large end, but on a smaller scale—not a portion of the exact outline at the large end, as produced by the formed tool process and as shown in the figure. The method of adjusting the cuts to approximate the desired shape is described in the next chapter.

The Templet Principle: This principle is illustrated in Fig. 28, in skeleton form only. A former or templet is used which has the same outline as would a tooth of the gear being cut, if the latter were extended as far from the apex of the pitch cone as the position in which the former is placed. The tool is carried by a slide which reciprocates it back and forth along the length of the tooth in a line of direction (OX, OY, etc.) which passes through vertex O of the pitch cone. This slide may be swiveled in any direction and in any plane about this vertex, and its outer end is supported by the roller on the former. With this arrangement, as the slide is swiveled inward about the vertex, the roll runs up on the templet, raising the slide and the tool so as to reproduce on the proper scale the outline of the former on the tooth being cut. Since the movement of the tool is always toward

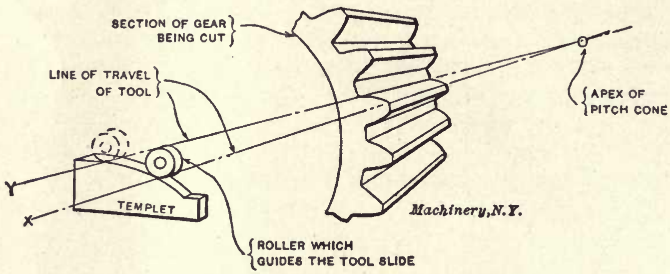


Fig. 28. Illustrating the Templet Principle for Forming the Teeth of Bevel Gears

the vertex of the pitch cone, the elements of the tooth vanish at this point and the outlines are similar at all sections of the tooth, though with a gradually decreasing scale as the vertex is approached—all as required for correct bevel gearing.

The arrangement thus shown diagrammatically is modified in various ways in different machines, but the movement imparted to the tool in relation to the work is the same in all cases where the templet principle is employed, no matter what the connection between the templet and the tool may be.

The Mold-generating Principle: Suppose we have a bevel gear blank made of some plastic material, such as clay or putty. By transposing Formula 34, $\sin a_p = \dfrac{N_p}{N_g}$ to read $N_g = \dfrac{N_p}{a_p}$, it is evidently possible to make a crown gear which will mesh properly with any bevel gear, such as the one we wish to form. If this crown gear and the plastic blank are properly mounted with relation to each other and rolled together, the tooth of the crown gear will form tooth spaces and teeth of the proper shape in the blank. This is the foundation principle of the molding-generating method.

In practice we have blanks of solid steel or iron to machine instead of putty or clay, so the operation has to be modified accordingly. Fig.

29 shows in diagrammatic form an apparatus for using the shaping or planing operation with the molding-generating principle. Here the crown gear is of larger diameter than is required to mesh with the gear being cut, and it engages a master gear keyed to the same shaft as the gear being cut, and formed on the same pitch cone. If the teeth of the crown gear, instead of being comparatively narrow as shown, were extended clear to the vertex *O*, they would mesh properly with the gear to be cut. The tooth is provided as shown having a line of movement such that the point of the tooth travels in line *OX*, which is the corner of a tooth of an imaginary extension of the crown gear. This crown gear has a plane face (see reference to "octoid" form of tooth on page 20) and the cutting edge of the tooth is straight and

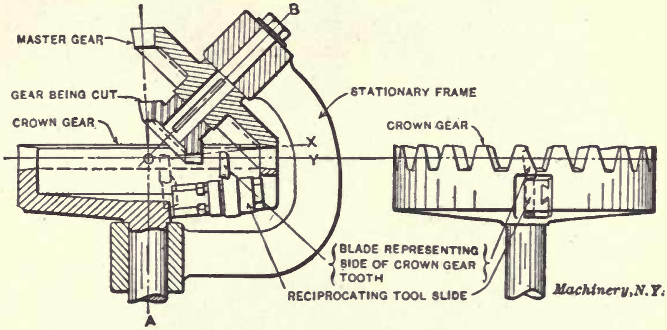


Fig. 29. Model Illustrating the Planing or Shaping Operation Applied to the Molding-generating Principle of Forming Teeth of Bevel Gears

set to mesh the face of the tooth. As it is reciprocated by suitable mechanism (not shown) the cutting edge represents a face of the imaginary crown gear tooth. If now, the master gear and crown gear are rolled together and the tool reciprocating starts in at one side of the gear to be cut and passing out at the other, the straight cutting edge of the tooth will generate one side of a tooth in the gear to be cut in the same way as if the extended tooth of the crown gear were rolling its shape on one side of the tooth of a plastic blank. This simple mechanism has, of course, to be complicated by provisions for cutting both sides of the tooth, and for indexing the work from one tooth to the other so as to complete the entire gear. Arrangements have to be made also to make the machine adjustable for bevel gears of all angles, numbers of teeth and diameters within its range.

The use of the three principles illustrated in Figs. 27, 28 and 29 is not limited to the cutting operation shown for each case. In Fig. 27, for instance, a formed planer or shaper tool may be used as well as a formed milling cutter. Templet machines have been made in which a milling cutter is used instead of a shaper tool. This is true also of the molding-generating principles shown in Fig. 29.

Machines for Cutting the Teeth of Bevel Gears by the Formed Tool Process

A very common method of using the formed tool for cutting bevel gears makes use of the ordinary plain or universal milling machine and adjustable dividing head. Cutting bevel gears by this method is described in the next chapter, so it will not be described here.

Most builders of automatic gear cutting machines furnish them, if desired, in a style which permits the swiveling of the cutter slide or of the work spindle to any angle from 0 to 90 degrees, thus permitting the automatic cutting of bevel gears by the formed cutter process.

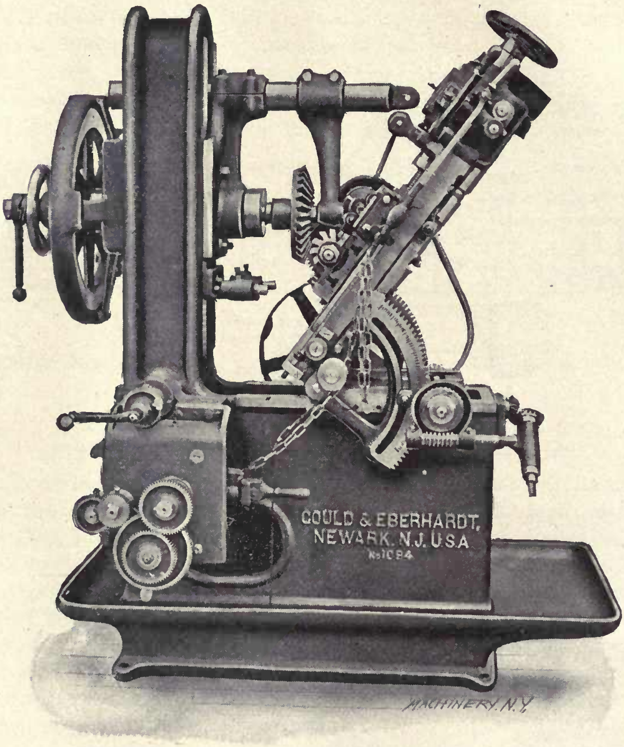


Fig. 30. Gould and Eberhardt Automatic Machine Cutting a Bevel Gear

An example of such a machine is shown in Fig. 30. Here the cutter slide is mounted on an adjustable swinging sector, as may be seen. As explained in the next chapter, it is necessary when cutting bevel gears, to cut first one side of the teeth all around and then the other. Between the two cuts the relation of the work and cutter to each other, as measured in a direction parallel to the axis of the cutter spindle, has to be altered. In the automatic machine this is effected by shifting the cutter spindle axially when the second cut around on the other side of the teeth is taken. Suitable graduations are provided for the angular and longitudinal adjustments.

Bevel Gear Templet Planing Machines

The templet planing machine most commonly used in this country is shown in one of the smaller sizes in Fig. 31. The tool is carried by a holder reciprocated by an adjustable, quick-return crank motion. The slide which carries this tool-holder may be swung in a vertical plane about the horizontal axis on which it is pivoted to the head, which carries the whole mechanism of tool-holder, slide, crank, driving gearing, etc. This head, in turn, may be swung in a vertical axis about a pivot in the bed. The circular ways which guide this movement are easily seen in the illustration. The intersection of the vertical and horizontal axes of adjustment (which takes place in mid-air in front of the tool-slide) is the point *O* in Fig. 28 where the templet

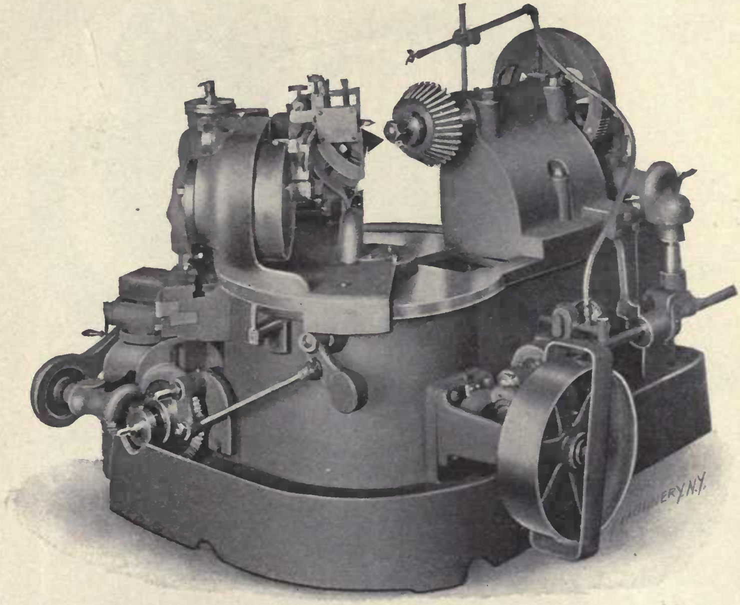


Fig. 31. Gleason Templet-controlled Bevel-gear Planing Machine

principle is shown in diagrammatic form. The blank is mounted on a spindle carried by a head which is adjustable in and on the top of the bed of the machine so that the apex of the cone of the gear may be brought to point *O* by means of the gages which are a part of the equipment of the machine.

Three templets are used, mounted in a holder attached to the front of the bed, on the further side in the view shown. The first of these templets is for "stocking" or roughing out the tooth spaces. It guides the tool to cut a straight gash in each tooth space, removing most of the stock. After each tooth space has been gashed in this fashion, the templet holder is revolved to bring one of the formed templets into position, and a tool is set in the holder so that its point bears the same relation to the shape of the tooth desired as the cam roll does to the templet. The head is again fed in by swinging it around its vertical axis, during which movement the roll runs up on the stationary templet, swinging the tool about its horizontal axis in such a way as to duplicate the desired form on the tooth of the gear. One side of each tooth being thus shaped entirely around, the holder is again revolved to bring the third templet into position. This has a reverse form from the preceding one adapted to cutting the other side of the tooth. A tool with a cutting point facing the other way being inserted in the holder, each tooth of the gear has its second side formed automatically, as before, completing the gear. The swinging

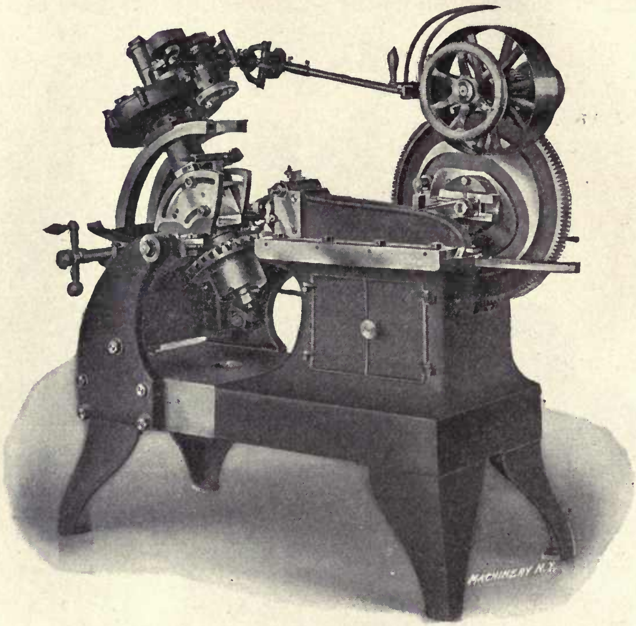


Fig. 32. The Bilgram Bevel Gear Generating Machine

movement for feeding the tool and the indexing of the work are taken care of by the mechanism of the machine without attention on the part of the operator.

Bevel Gear Generating Machines

The mechanism illustrated in outline in Fig. 29 is one that has been employed in a number of interesting and ingenious machines. The first application of this principle was made by Mr. Hugo Bilgram of Philadelphia, Pa. This form of machine in the hand-operated style has been used for many years. An example of a more recently developed automatic machine of the same type is shown in Fig. 32. The movements operate on the same principle as in Fig. 29, though in a modified form. Instead of rotating the crown gear and master gear together, the imaginary crown gear and, consequently, the tool, remain stationary so far as angular position is concerned, while the frame is rotated about the axis of the crown gear, thus rolling the master gear on the latter and rolling the work in proper relation to the tool. Instead of using crown and master gears, however, a section of the pitch cone of the master gear is used, which rolls on a plane surface, representing the pitch surface of the crown gear. The two surfaces are prevented from slipping on each other by a pair of steel tapes, stretched so as to make the movement positive. A still further change consists in extending the work arbor down beyond center *O* in Fig. 29, mounting the blank on the lower side of the center so that the tool, being also on the lower side, is turned the other side up from that shown in the diagram. All these movements can be followed in Fig. 32. As explained, a tool with a straight edge is used, representing

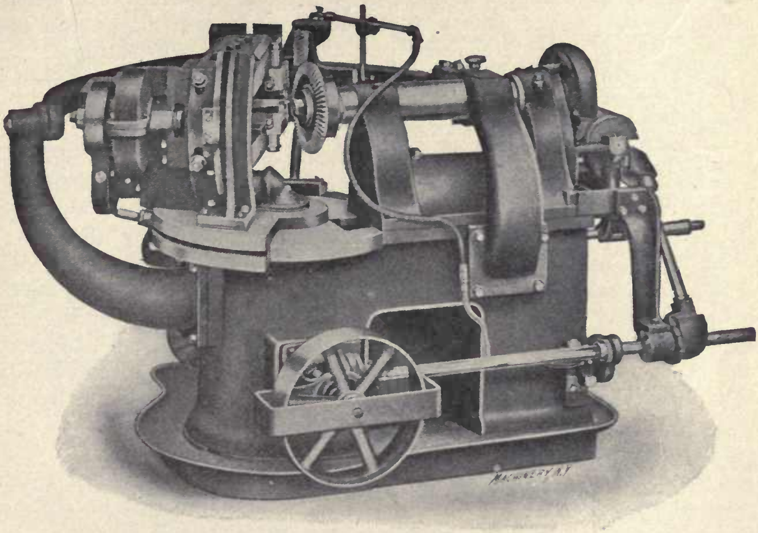


Fig. 33. Gleason Bevel Gear Generating Machine

the side of a rack tooth, and this tool is reciprocated by a slotted crank, adjustable to vary the length of the stroke, and driven by a Whitworth quick-return movement. The feed of the machine is effected by swinging the frame in which the work spindle and its supports are hung, about the vertical axis of the imaginary crown gear.

As stated, the machine is automatic. The operator sets the machine and places a previously-gashed blank on the work spindle and starts the tool in operation. The mechanism provided will, without further attention, complete one side of all the teeth. The machine may be then readjusted and the tool set for cutting the other side, which will be finished in the same automatic fashion. The mechanism does not operate on the principle of completing one side of one tooth before going to the next. It follows the plan of indexing the work for each stroke of the tool, the rolling action being progressive with the indexing so as to finish all the teeth at once.

The Gleason generating machine is shown in Fig. 33. It differs from the previous machine in employing two tools, one on each side of the tooth. The construction is identical with the mechanism in Fig. 29, in having the axes of the tool-slides and of the blank fixed in relation to each other during the operation, the tool-holders and the blank rocking about their axes to give the rolling movement for cutting. The rocking is effected by means of segments of an actual crown gear and master gear. The segment of the crown gear is permanently attached to the face of the rear of the cutter slide frame, while the segment of the master gear (of which there are several furnished with the machine, the one used being chosen to agree with the angle of the gear to be cut) is clamped to the semi-circular arm pivoted at the outer end of the machine at one side, and fastened to the work spindle sleeve on the other. This arm is rocked by a cam mechanism and slotted link on the side opposite that shown in the illustration.

The cycle of operations is as follows: The machine being adjusted properly in its preliminary position, the tool-slide and the head on which it is mounted are swung back about the vertical axis so that the tools clear the work. The blank being set in the proper position, a cam movement swings the cutter slide head inward until the reciprocating tools reach the proper depth. The cam movement first mentioned now rocks upward the semi-circular arm extending around the front of the machine, rolling the blank and (through the segmental crown and master gears) the slide, until the tools have been rolled out of contact in one direction, partially forming the teeth as they do so. The arm is then rolled back to the central position and along downward to the lower position, until the tools are rolled out of contact with the tooth in this direction, completing the forming of the proper shape as they do so. The cam then rocks the arm back to the central position, where the cutter-slide head is swung back to clear the tooth, and the work is indexed, after which this cycle of operations is continued for the next tooth. It will be seen that by starting from the central position, going to each extreme and returning, all parts of each tooth are passed over twice, giving a roughing and a finishing chip. The machine is entirely automatic.

CHAPTER VII

CUTTING THE TEETH OF BEVEL GEARS

Special directions for operating are furnished by the makers of molding-generating and templet planing machines. As these directions are usually adequate, and apply only to the particular machines for which they are given, this chapter will be confined to giving instructions for cutting teeth by the formed tool method only, as performed on standard machine tools.

The Practicability of the Formed Tool Process

The first piece of instruction to be given in cutting bevel gears with a milling cutter is—don't do it. There are exceptions a-plenty to this rule, of course. For instance, gears too small to be cut on any commercial planing machine may be milled with a formed cutter; in general, it is not considered advisable to plane gears having teeth finer than 12 to 16 diametral pitch. It is allowable, also, to mill gears of coarser pitch which are to run at slow speeds or which are to be used only occasionally—such, for instance, as the bevel gears used for driving the elevating screws of a planer cross-rail, or those used in connection with any hand-operated mechanism. It is impracticable under ordinary conditions to mill teeth of bevel gears having teeth coarser than 3 diametral pitch, no matter what the service for which they are to be used.

Cutting Bevel Gears in the Milling Machine

The first requirement for setting up the milling machine to cut bevel gears is a true-running blank, with accurate angles and diameters. If such a blank cannot be found in the lot of gears to be cut, it will be necessary to turn up a dummy out of wood or other easily worked material. Otherwise the workman is inviting trouble, whatever his method of setting up.

Fig. 34 shows the machine set up for cutting a bevel gear, and Fig. 35 shows in diagram form the relative positions of the cutter and the work. The spindle of the dividing head is set at the cutting angle, as shown, and the cutter (which has been centered with the axis of the work-spindle) is sunk into the work to the whole depth W, as given by the working drawing.

The Brown & Sharpe Mfg. Co. recommends that for shaping with a formed cutter, the cutting angle be determined by subtracting the *addendum* angle from the pitch cone angle, instead of subtracting the dedendum angle as in Rule 15. In other words, the clearance at the bottom of the tooth is made uniform, as shown in Fig. 37, instead of tapering toward the vertex. This gives a somewhat closer approximation to the desired shape.

The centering may be done by mounting a true hardened center in the taper hole of the spindle, and lining up its point with the mark which will be found inscribed either on the top or on the back face of the tooth of the commercial gear cutter. A more accurate method is described in MACHINERY'S Shop Operation Sheet, No. 1. Setting the cutter to the whole depth *W* is effected by passing the work back and forth under the revolving cutter and slowly raising it until the teeth of the cutter just bite a piece of tissue paper laid over the edge of the blank. This must be done after centering. The dial on the elevating screw shaft is set at zero in this position, and then the knee is raised an amount equal to the whole depth of the tooth, reading the dial from zero. This is evidently not exactly right, since the measurement should be taken in the direction of the back edge of the tooth, which inclines from the perpendicular an amount equal to the dedendum angle, as

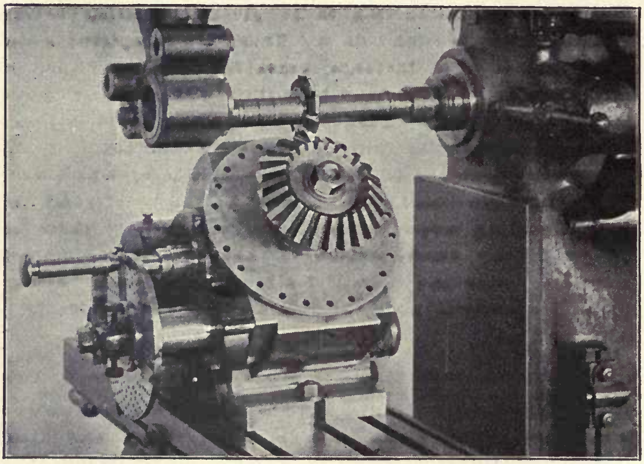


Fig. 34. Milling Machine Set Up for Cutting a Bevel Gear

shown in Fig. 35. In practice, the slight difference in the value for the whole depth thus obtained is negligible.

Having thus mounted the work at the proper angle and having thus centered the cutter and set it to depth, two tooth spaces should next be cut, with the indexing set by the tables furnished with the dividing head to give the number of teeth required for the gear. Cutting these two spaces leaves a tooth between on which trial cuts are to be made until the desired setting is obtained. The relative positions of the cutter and the work and the shape of the cuts thus produced are shown in the upper part of Fig. 35. It will be seen at once that this does not cut the proper shape of tooth. As explained in the first paragraph in Chapter VI, all the elements of the bevel gear tooth vanish at *O*, the vertex of the pitch cone—that is to say, the outer corners of the tooth space should converge at *O* instead of at *A*, and the sides of the tooth spaces at the bottom, instead of having the parallel width given them by the formed cutter, should likewise vanish at *O*. Our next problem is that of so re-setting the machine that we can cut gear teeth as nearly as possible like the true tooth-form in which the elements converge at *O*.

Offsetting and Rolling the Blank to Approximate the Shape of Tooth

There are a number of ways of approximating the desired shape of bevel gear teeth. Of these we have selected as most practicable the one

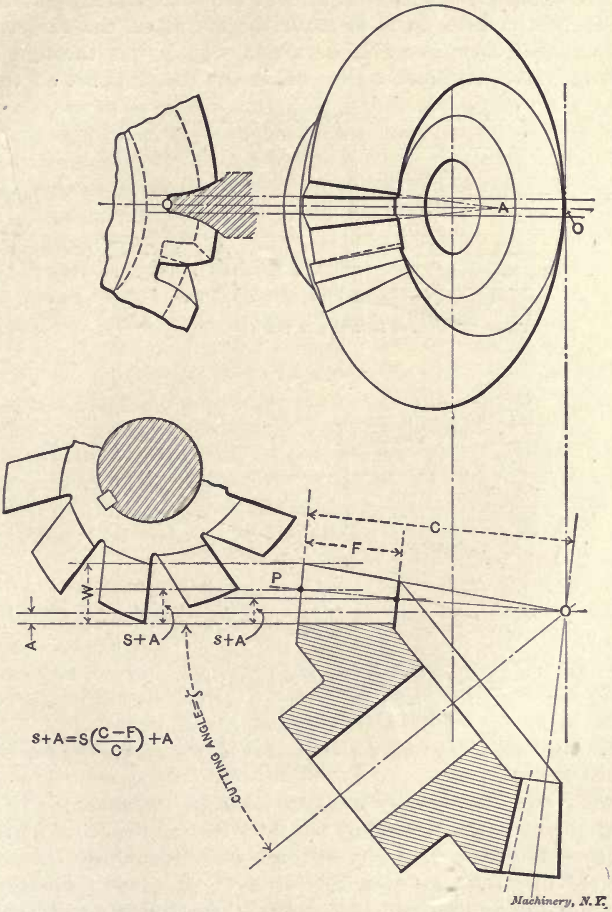


Fig. 35. Relative Positions of the Formed Cutter and the Blank when taking a Central Cut

in which the sides of the tooth at the pitch line converge properly toward the vertex of the pitch cone. Gears cut by this process will show, of course, the proper thickness at the pitch line when measured by the gear tooth caliper at either the large or the small ends. This method of approximation produces tooth spaces which, at the small end, are somewhat too wide at the bottom and too narrow at the top, or, in other words, the teeth themselves at the small end are too narrow at the bottom and too wide at the top. To make good running gears they must be filed afterward by hand, as described later. When so filed they are better than milled gears cut by other methods of approximation which omit the hand filing.

In the upper part of Fig. 36 is shown a section of the gear in Fig. 35, taken along the pitch cone at *PO*. It will be seen that the teeth at the pitch line converge, but meet at a point considerably beyond the vertex *O*. What we have to do is to move the cutter off the center, so that it will cut a groove, one side of which would pass through *O* if extended that far. The amount by which the cutter is set off the center

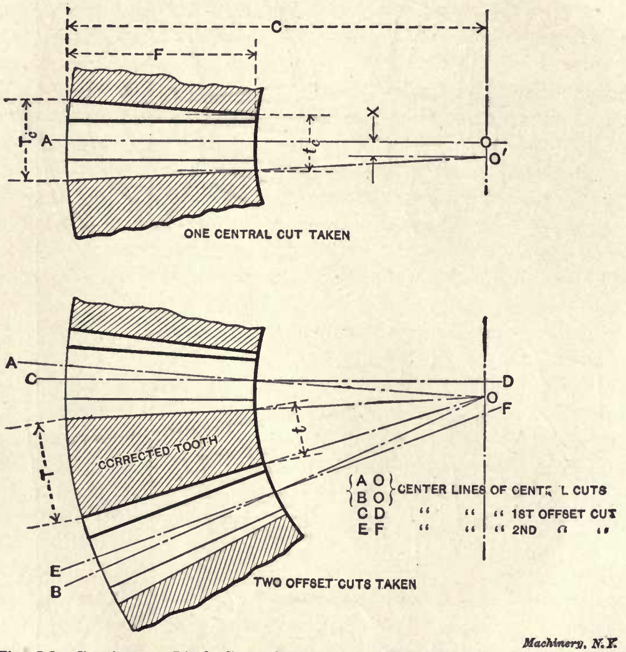


Fig. 36. Section on Pitch Cone Surface PO of Fig. 35, showing Central and Offset Cuts

is known as the "set-over." We may take, for instance, for trial a set-over equal to 5 or 6 per cent of the thickness of the tooth at the large end. Move the face of the trial tooth away from the cutter by the amount of this trial set-over, having first, of course, run the cutter back out of the tooth space. Now rotate the dividing head spindle to bring this tooth face back to the cutter again, stopping it where the cutter will about match with the inner end of the space previously cut. Take a cut through in this position.

Next index the work to bring the cutter into the second tooth space and move the blank over to a position the other side of the central position by an amount equal to the same set-over, thus moving the opposite face of the trial tooth away from the cutter. Rotate the dividing head spindle again to bring this face toward the cutter until the latter matches the central space already cut at the inner end of the teeth. Take the cut through in this position.

Now with vernier gear tooth calipers or with fixed gages machined to the proper dimensions measure the thickness of the tooth at the pitch line at both large and small ends (the values for the addendum and the thickness of the pitch line at both ends of the tooth are given by Rules 5, 8, 10 and 11). If the thickness is too great at both the large and the small ends, rotate the tooth against the cutter and take another cut until the proper thickness at either the large or small end has been obtained. If the thickness comes right at both ends the amount of set-over is correct. If it is right at the large end and too thick at the small end, the set-over is too much. If it is right at the small end and too thick at the large end the set-over is not enough. The recommended trial set-over (5 or 6 per cent of thickness of the tooth at the pitch line at the large end) will probably not be enough, so two or three cuts will have to be taken on each side of the trial tooth, as described, before the proper amount is found.

Having found the proper set-over, the cross-feed screw is set to that amount and the cut is taken clear around the gear. Then the cross-feed screw is set to give the same amount of set-over the other side of the center line and the work is rotated until the cutter matches the tooth spaces already cut at the small end and is run through the work. The tooth will generally be found too thick, so the work spindle is rotated still more until the tooth is of the proper thickness, when the gear is again cut clear around on this second cut.

The number of holes it was necessary to move the index pin on the dividing plate circle between the first and the second cuts to get the proper thickness of tooth, should be recorded. On succeeding gears it will thus only be necessary to take a first cut clear around with the work set over by the required amount on one side of the center line, and then a second cut around with the work set over on the other side of the center line, rotating the index crank the number of holes necessary to give the proper thickness of tooth between the cuts.

It will be noted that the shifting of the blank by the index crank is only used for bringing the thickness of tooth to the proper dimension. In some cases, particularly in gears of fine pitch and large diameter, this adjustment will not be fine enough—that is to say, one hole in the index circle will give too thick a tooth and the next one too thin a tooth. To subdivide the space between the holes, most dividing heads have a fine adjustment for rotating the worm independently of the crank. Every milling machine should be provided with such an adjustment.

In large gears it is best to take the central cuts shown in Fig. 35 clear around every blank before proceeding with the approximate cuts. This gives the effect of roughing and finishing cuts, and produces more accurate gears. The central cuts may be made in a separate operation with a roughing or stocking cutter if desired. It might also be mentioned that it is common practice to turn up a wooden blank for making the trial cuts shown in Figs. 35 and 36, to avoid the danger of spoiling the work by mistakes in the cut-and-try process.

Positive Determination of the Set-over*

This cut-and-try process, however, may be practically eliminated by calculating the set-over from the following table and formula:

TABLE FOR OBTAINING SET-OVER FOR CUTTING BEVEL GEARS

No. of Cutter	Ratio of Pitch Cone Radius to Width of Face $\left(\frac{C}{F}\right)$												
	$\frac{3}{1}$	$\frac{3\frac{1}{4}}{1}$	$\frac{3\frac{1}{2}}{1}$	$\frac{3\frac{3}{4}}{1}$	$\frac{4}{1}$	$\frac{4\frac{1}{4}}{1}$	$\frac{4\frac{1}{2}}{1}$	$\frac{4\frac{3}{4}}{1}$	$\frac{5}{1}$	$\frac{5\frac{1}{2}}{1}$	$\frac{6}{1}$	$\frac{7}{1}$	$\frac{8}{1}$
1	.254	.254	.255	.256	.257	.257	.257	.258	.258	.259	.260	.262	.264
2	.266	.268	.271	.272	.273	.274	.274	.275	.277	.279	.280	.283	.284
3	.266	.268	.271	.273	.275	.278	.280	.282	.283	.286	.287	.290	.292
4	.275	.280	.285	.287	.291	.293	.296	.298	.298	.302	.305	.308	.311
5	.280	.285	.290	.293	.295	.296	.298	.300	.302	.307	.309	.313	.315
6	.311	.318	.323	.328	.330	.334	.337	.340	.343	.348	.352	.356	.362
7	.289	.298	.308	.316	.324	.329	.334	.338	.343	.350	.360	.370	.376
8	.275	.286	.296	.309	.319	.331	.338	.344	.352	.361	.368	.380	.386

For obtaining the set-over by the above table, use this formula:

$$\text{Set-over} = \frac{T_c}{2} - \frac{\text{factor from table}}{P}$$

P = diametral pitch of gear to be cut,

T_c = thickness of cutter used, measured at pitch line.

Given as a rule this would read: *Find the factor in the table corresponding to the number of the cutter used and to the ratio of pitch cone radius to width of face; divide this factor by the diametral pitch, and subtract the result from half of the thickness of the cutter at the pitch line.*

As an illustration of the use of this table in obtaining the set-over we will take the following example: A bevel gear of 24 teeth, 6 pitch, 30 degrees pitch cone angle and 1¼ face. These dimensions, by the rules given in Chapter I, call for a No. 4 cutter and a pitch cone radius of 4 inches.

In order to get our factor from the table, we have to know the ratio of pitch cone radius to width of face. This ratio is $\frac{4}{1.25} = \frac{3.2}{1}$ or about $\frac{3\frac{1}{4}}{1}$. The factor in the table for this ratio with a No. 4 cutter is 0.280. We next measure the cutter at the proper depth of $S + A$ for 6 pitch, which is found in the column marked "depth of space below pitch line" in a regular table of tooth parts, or by dividing 1.157 by the diametral pitch. This gives $S + A = 0.1928$ inch. We find, for in-

* MACHINERY, December, 1909.

stance, that the thickness of the cutter at this depth is 0.1745 inch. The dimension will vary with different cutters, and will vary in the same cutter as it is ground away, since formed bevel gear cutters are commonly provided with side relief. Substituting these values in the formula, we have

$$\text{Set-over} = \frac{0.1745}{2} - \frac{0.280}{6} = 0.0406''.$$

which is the required dimension.

With reference to the use of the above table and formula, the Brown & Sharpe Mfg. Co., after trial in its gear-cutting department, says: "We feel fairly confident it is within working limits of being satisfactory." While this sounds encouraging, it will evidently be wise to be

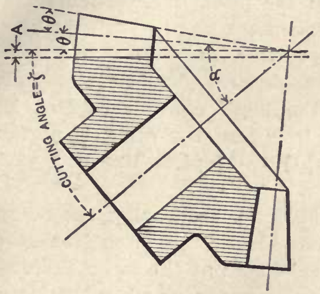
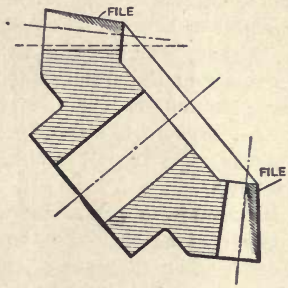


Fig. 37. Cutting Angle and Parallel Clearance Recommended by Brown& Sharpe for Cutting with Formed Cutter



Machinery, N.Y.

Fig. 38. The Surfaces to be Filed, in Fitting Bevel Gears Cut with a Formed Cutter

sure we are right before going ahead. So the trial tooth should be measured, the same as when the cut-and-try process is used.

Use of the Formula for Other Methods of Correction

It is customary also among workmen expert in cutting bevel gears with formed cutters, to cut loose from rules and formulas for the selection of the cutters, and depend on their experience to get shapes which require somewhat less filing than would otherwise be necessary. Whenever this "cutting loose" requires, as it sometimes does, the use of a cutter of finer pitch than that of the teeth of the bevel gear at the large end, the values given in the table are inapplicable. The following formula may then be used:

$$\text{Set-over} = \frac{T_c}{2} - \frac{T_c - t_c}{2} \times \frac{C}{F}$$

in which t_c is the thickness of the cutter measured at a depth $s + A$, obtained as shown in Fig. 35. This has been tried on several widely varying cases with good results. It requires, it will be seen, two measurements of the cutter in place of the single one required when the regular pitch of cutter is used.

Filing the Teeth

The method of cutting bevel gears just described requires the filing of the points of the teeth at the small end. This can be done "by the eye" very skillfully when the workman is used to it. The operation consists in filing off a triangular area extending from the point of the tooth at the large end to the point at the small end, thence down to the pitch line at the small end and back diagonally to the point at the large end again. This is shown in Fig. 38 by the shaded outline. Enough is taken off at the small end of the tooth so that the edges of the teeth at the top appear to converge at vertex *O*.

The bevel gears may be tested for the accuracy of the cutting and filing by mounting them in place in the machine and revolving them at high speed, or by mounting them in a testing machine made for the purpose. The marks of wear produced by running them together under pressure, with the back faces flush with each other, should extend the whole length of the tooth at the pitch line. If it does not, the amount of set-over allowed in cutting them was at fault, being too little if they bear heavily at the large ends, and too much if they bear heavily at the small ends. The bearing area should also be fairly evenly distributed over the sides of the teeth above the pitch line, from the large to the small end. If it is not, the filing is at fault. The marks of wear will not extend far below the pitch line in a pinion of few teeth.

It is possible to get along without filing by decreasing the amount of set-over so as to make the teeth too thin at the pitch line at the small end, when they are of the right thickness at the large end. This does not give quite as good running gears, however, as when the method just described is followed.

Cutting Bevel Gears on the Automatic Gear-cutting Machine

The directions for cutting bevel gears on the milling machine apply in modified form to the automatic gear-cutting machine as well. The set-over is determined in the same way, but instead of moving the work off center, the cutter spindle is adjusted axially by means provided for that purpose. Some machines are provided with dials for reading this movement. The cutter is first centered as in the milling machine, and then shifted—first to the right, and then to the left of this central position.

The rotating of the work to obtain the proper thickness of tooth is effected by unclutching the indexing worm from its shaft (means usually being provided for this purpose) and rotating the worm until the gear is brought to proper position. Otherwise the operations are the same as for the milling machine.